MATH2011 Assignment 5, MATH2100 Assignment 10

2 1. Find the steady-state temperature distribution $u(x,y)$ in the square $0 \leq x \leq 2,\ 0 \leq y \leq 2$ if the lower side is kept at a temperature $u = \sin\frac{\pi x}{2}$ and $u = 0$ on the other three sides.

2 2. Find the steady-state temperature $u(x,y)$ in the strip $0 \leq x \leq \pi$, $y > 0$ with the vertical sides perfectly insulated, $u(x,y)$ bounded as $y \to \infty$ and the lower side kept at temperature x. (*Do from first principles, don't use Laplace transform!*)

2 3. Kreyszig Set 4.5, p.226, Q.4.

4 4. Kreyszig Set 11.10, p.635, Q11; Q12 (BONUS), Q13; Q14 (BONUS).

Q4 p226

$4x^2 y'' + 4xy' + (x - \frac{1}{36})y = 0, \quad z = \sqrt{x}$

$y' = dy/dz \cdot \frac{1/2}{z}$

$y'' = \frac{1}{2}\frac{d}{dz}\left(\frac{dy}{dz}\cdot\frac{1}{z}\right)\frac{1}{2z}$

$= \frac{1}{4z^2}\frac{d^2y}{dz^2} - \frac{1}{4z^3}dy/dz$

$4z^4\left(\frac{1}{4z^2}\frac{d^2y}{dz^2} - \frac{1}{4z^3}dy/dz\right) + 4z^2\frac{1}{2z}\frac{1}{2z}\frac{dy}{dz} + (z^2 - \frac{1}{36})y = 0$

$z^2\frac{d^2y}{dz^2} + z\,dy/dz + (z^2 - 1/36)y = 0$

$y(z) = a J_{1/6}(z) + b J_{-1/6}(z)$