Tail-adaptive financial modeling with the GSHD

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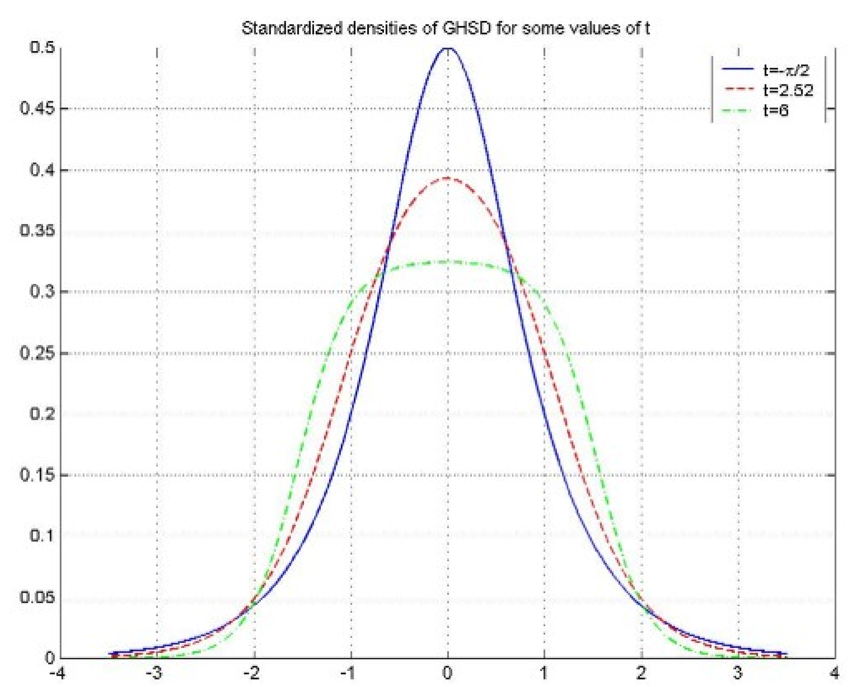
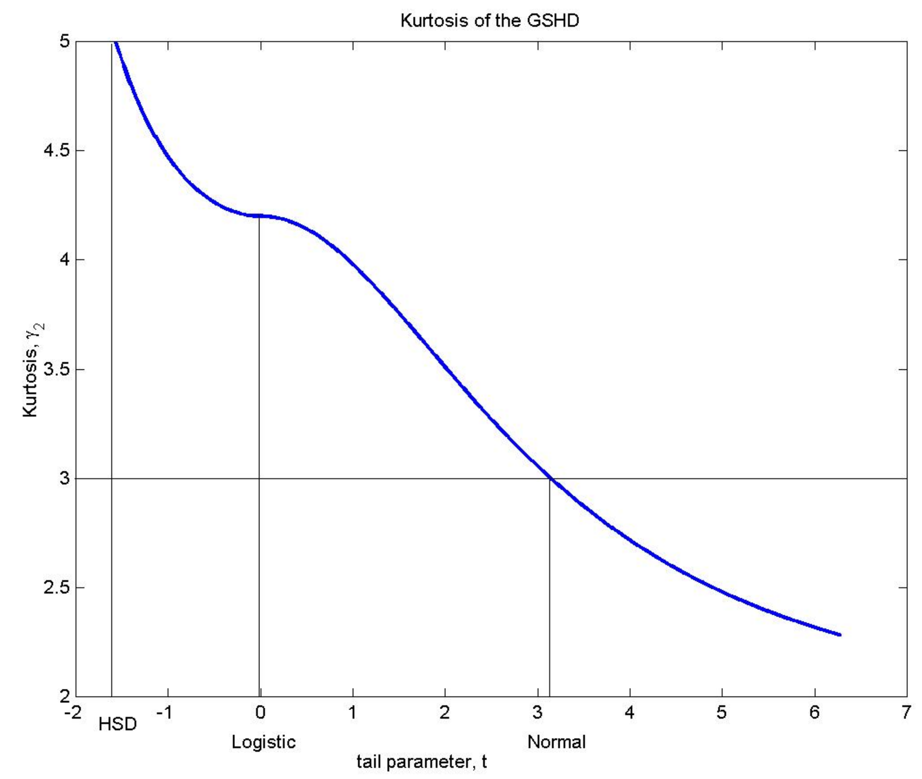
Abstract

The generalized secant hyperbolic distribution (GSHD) has been studied recently as a modeling tool in financial data analysis. The GSHD is completely specified by location, scale and shape parameters. We demonstrate that the shape parameter may be understood as a tail weight parameter of the distribution, and introduce a three-class classification procedure based on various estimators of the tail weight of the GSHD. We illustrate the classification with large-sample examples of financial applications.

Introduction

The GSHD is a location-scale family of unimodal symmetric distributions that includes the Cauchy and the uniform distributions as its limiting heavy-tail and light-tail cases. A member of the distribution is completely specified by the location,μ, scale,σ, and shape, t, parameters.The GSHD is a promising modeling tool for asset returns. Several computationally attractive estimators of location are available for this distribution for the location (or regression) problem. These estimators retain a high efficiency within wide ranges of the shape parameter. It is thus possible to introduce an adaptive estimation procedure based on a good shape classifier. It was shown elsewhere that the shape parameter may be understood as a tail weight parameter.





Tail classifiers

We suggest using the Hogg's, T, and Brys's, LQW, tail classifiers:

$$T = \frac{X_{0.975} - X_{0.025}}{X_{0.875} - X_{0.125}}, \qquad LQW_{0.125} = -\frac{X_{(1-0.125)/2} + X_{0.125/2} - 2X_{0.25}}{X_{(1-0.125)/2} - X_{0.125/2}},$$

where $X_{(\cdot)}$ are the sample percentiles.

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Adaptive procedures

A selection procedure may be based on either the T or LQW estimator or on a combination of both.

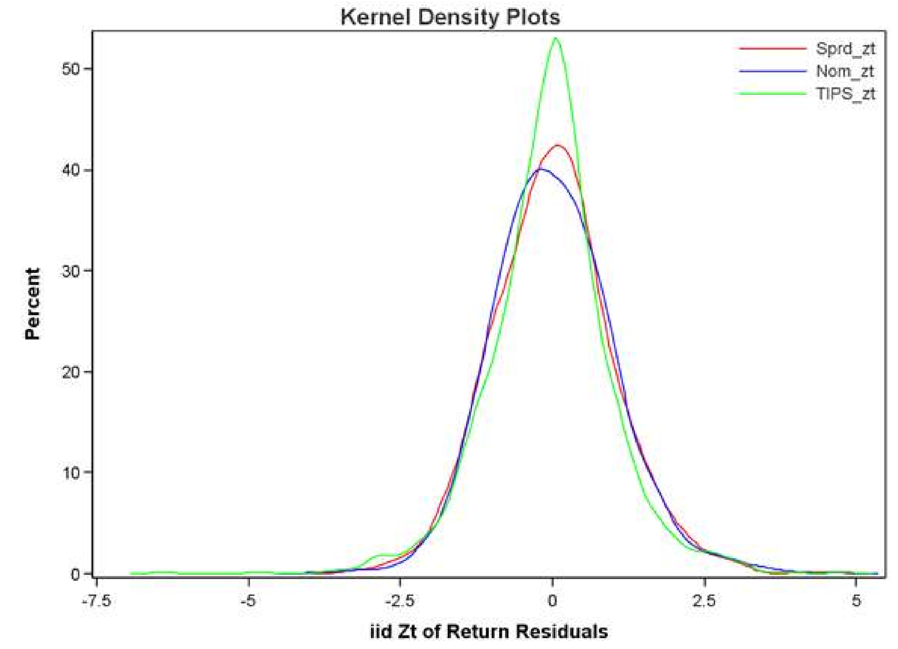
$T \geq 2.035\,(LQW_{0.125} \geq 0.352)$, heavy-tailed, i.e. $-\pi < t \leq -\pi/2$,

$1.467 < T < 2.035\,(0.142 < LQW_{0.125} < 0.352)$, normal-tailed, i.e. $-\pi/2 < t \leq 2\pi\,(-\pi/2 < t \leq 3\pi/2)$,

$T \leq 1.467\,(LQW_{0.125} \leq 0.142)$, light-tailed, i.e. $t > 2\pi\,(t > 3\pi/2)$.

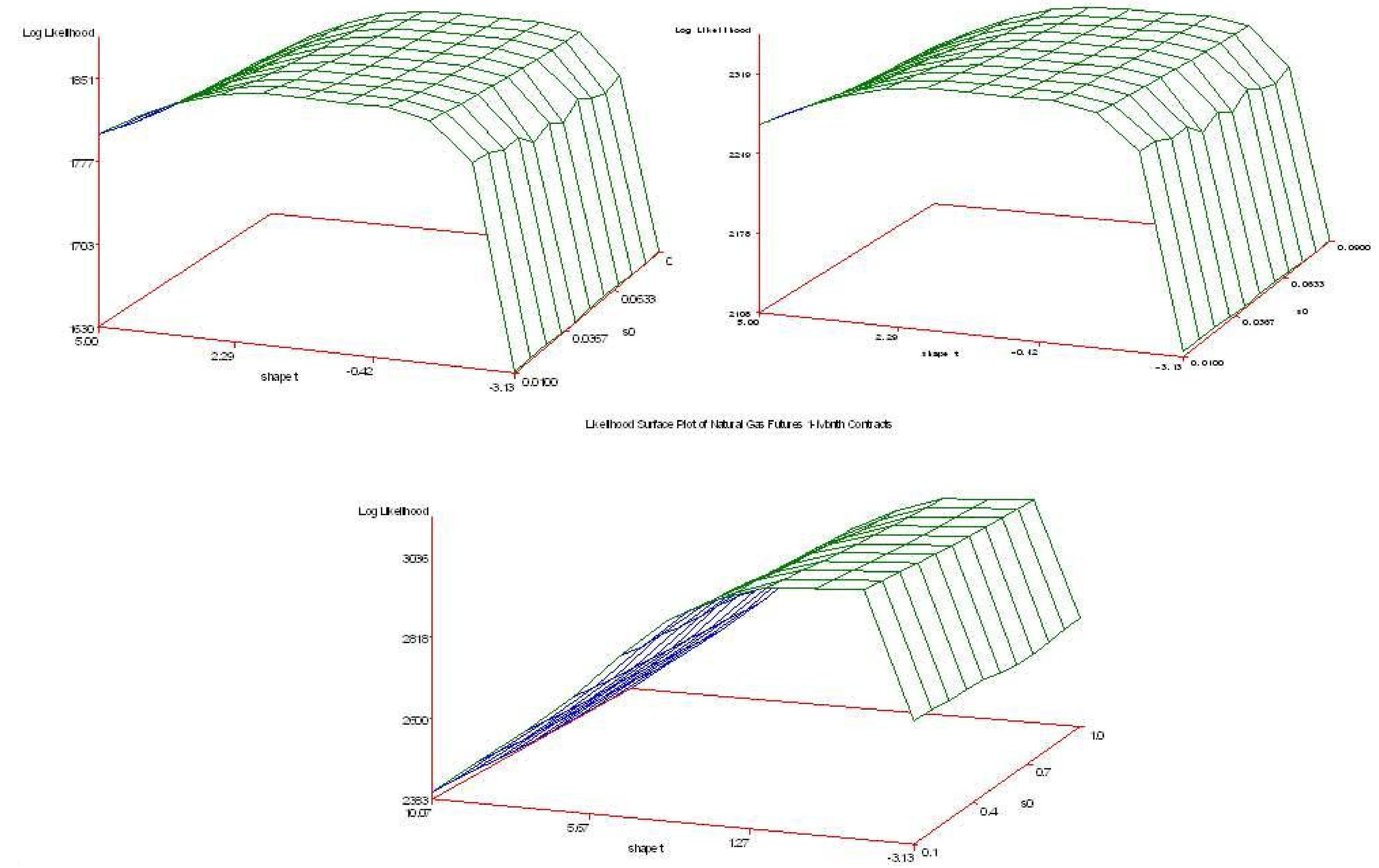
Finance series example

GARCH(1,1) residuals are assumed to follow the GSHD.



Asset Return	n	t_{MLE}
Natural Gas 1-month Ftr	3060	-1.63159
Natural Gas 2-month Ftr	3060	-1.53159
Natural Gas 3-month Ftr	3060	-1.93159
2008 Spread	1393	1.4384
2008 Nom	1434	1.498
2008 TIPS	1393	-1.9116

Likelihood surface



Conclusions

The GSHD may be a relevant distributional assumption for finance econometric models. Its flexibility in capturing heterogenous shapes of asset returns is very attractive. For instance, very heavy-tailed data such as those that come from a Cauchy distribution can be approximated by the GSHD, which has the advantage that all moments are finite. A possible direction for our future research is testing nonzero skewness detected in the empirical portion of our current study.

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