Simple unbounded integrand

The integrand f in this example is simple but unbounded and hence not Riemann integrable. For $x = 2^{-n}$ the function value $f(x) = n + 1$, otherwise $f(x) = 0$. A suitable gauge would be equal to $\dfrac{\epsilon}{(2^n)(n+1)}$ if $x = 1/2^n$ and 1 otherwise. The definition of the Kurzweil inegral requires that for **all tagged divisions** which are gauge-fine the Riemann sum is within ε of the expected value of the integral. Unfortunately and inevitably the computer will display only one sum. With the above choice our program will produce 0 for the Riemann sum, a rather uninteresting result. Therefore we aim below for the worst possible scenario for the Riemann sum. For a good display we choose a fairly large ε. This might cause the contribution from the subinterval which is tagged by 1 becoming too large. In order to prevent this we make an adjustment in our definition of gauge. Our choice of ε below causes a fairly large error. However we are interested in a good graphical display rather than in accuracy. First we denote $n = \log_2(x)$ and then we define the integrand as

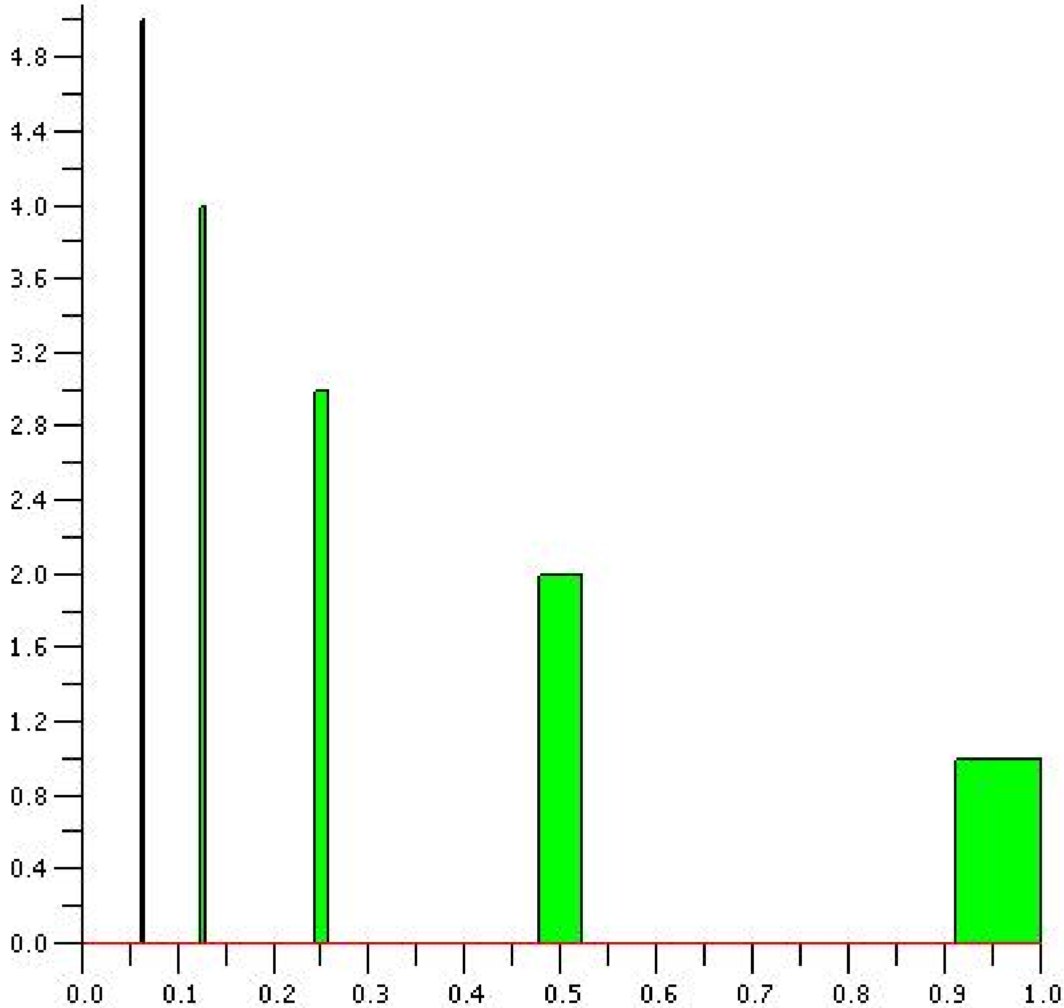
$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ (\lfloor n+1 \rfloor - \lceil n \rceil)(-n+1) & \text{otherwise.} \end{cases}$$

The gauge δ is defined as follows:

$$\delta(x) = \begin{cases} \eta & \text{if } x = 0 \\ \min\left(0.1,\ \lfloor n+1 \rfloor - \lceil n \rceil\right) \dfrac{x\varepsilon}{-n+1} + 0.9\left(2^{\lceil n \rceil} - x\right) & \text{otherwise.} \end{cases}$$

For the display we choose

$$\varepsilon = 0.9 \quad \text{and} \quad \eta = 0.05.$$



4.8
4.4
4.0
3.6
3.2
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