Graph Intersection problems and related trades

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A Survey of Open Graph Decomposition Problems

Brian Alspach

There has been considerable research on graph decomposition problems over the last forty years. I shall discuss some open problems in this area that I believe are important.

Diagonally cyclic latin squares

Nicholas Cavenagh

(Joint work with Carlo Hämäläinen and Adrian Nelson)

Like many combinatorial objects, diagonally cyclic latin squares have numerous, often unexpected, equivalences. This talk surveys these equivalences, as well as presenting an outline of a proof that any three cyclically generated transversals (of order prime $p > 7$) complete to a latin square.

Latin squares and biembeddings

Diane Donovan

In this talk I will review the concept of biembedding two latin squares of the same order. This presentation will be in terms of permutations on the rows, columns and elements of the squares. The talk will be concluded with the discussion of some open problems.

How to pull random regular graphs apart

Catherine Greenhill

As I understand it, people who work on graph decompositions enjoy pulling graphs apart. Usually they start with a fixed graph (which is often a complete graph or a complete bipartite graph) and then they break the graph up into the edge-disjoint union of subgraphs of specified types (such as cycles of given lengths).

This is reminiscent of some results in the theory of random regular graphs which describe how to pull random regular graphs apart. Here we start with a random regular graph and try to prove that, with high probability, it can be broken up into the edge-disjoint union of regular subgraphs of specified types (such as perfect matchings or Hamilton cycles). In fact, a stronger property known as contiguity is usually proved.

I will discuss contiguity of random regular graphs and the known results, giving all necessary definitions. Then I will consider whether exploration of the similarity between these two topics might lead to new contiguity results.

A linear algebraic approach to Latin squares and the spectrum of t-Latin trades

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To study orthogonal arrays and signed orthogonal arrays, Ray-Chaudhuri and Singhi (1988 and 1994) considered some module spaces. Using a linear algebraic approach we define an inclusion matrix and find its rank. In the special case of Latin squares we show that there is a straightforward algorithm for generating a basis for this matrix using the so-called intercalates. We also extend this last idea.

The concept of $t-(v,k)$ trade of block designs previously has been studied in detail. Also Latin trades has been studied in detail under different names. Khanban, Mahdian and Mahmoodian extended the concept of Latin trades and introduced $t-(v,k)$ Latin trades. Here we study the spectrum of these trades.

Counting small configurations in latin squares

Thomas McCourt

A partial triple system is a set, V, of v elements and a collection, $\mathcal{B}$, of triples, so that each unordered pair of elements occurs in at most λ triples of $\mathcal{B}$. A configuration is simply some partial triple system which typically has some fixed number of triples.

As a configuration is a partial triple system it is natural to consider larger combinatorial structures that may contain such a configuration. Indeed Grannell, Griggs and Mendelsohn did so for Steiner triple sustems. We consider the number of configurations contained in latin squares. Counting the configurations which contain at most four triples we find that the number of configurations containing at most three triples is constant for latin squares of a given order; however, for four triples there are five configurations that occur a constant number of times, 10 configurations that occur a variable number of times and one configuration that can not occur in a latin square. Similarly to the case for Steiner triple systems the number of occurences of these 11 configurations can be expressed in terms of the number of Pasch configurations (*intercalates*) present in a given latin square.

Degree/diameter Problem

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The well known *degree/diameter* problem for graphs (respectively, digraphs) is to determine the largest possible order $n_{d,k}$ of a graph (respectively, digraph), given degree (respectively, out-degree) at most $d \geq 1$ and diameter $k \geq 1$. There is a natural upper bound on the order of graphs (respectively, digraphs), given degree (respectively, out-degree) at most d and diameter k. There is a natural upper bound on $n_{d,k}$, given by

$$n_{d,k} \leq 1 + d + d(d-1) + \ldots + d(d-1)^{k-1}. \quad (1)$$

for undirected graphs, and

$$n_{d,k} \leq 1 + d + d^2 + \ldots + d^k. \quad (2)$$

for directed graphs (digraphs). The right-hand side is called the *Moore bound* for graphs (respectively, digraphs). If the equality sign holds then the graph (respectively, digraph) is called a *Moore graph* (respectively, *Moore digraph*). It is well known that Moore graphs and digraphs exist only for very few values of d and k.

In this talk we give an overview of Moore graphs and digraphs and the state of the art of the degree/diameter problem for both directed and undirected graphs.

The talk concludes with several open problems in this area.

Graph Labelling

Joe Ryan

Graph colouring is an assignment of colours to features of a graph (usually edges or vertices) subject to certain conditions, generally involving no two adjacent features being assigned the same colour. For some applications it becomes necessary to enforce an ordering on the assignment so colours are replaced by numbers. Once this step is taken, a number of algebraic operations and properties become available and the conditions constraining the labelling become richer. In this talk I will survey a number of graph labelling schemes looking at their origin, properties and possible applications. A number of conjectures and open problems will also be introduced.

Problems with Compuational Algorithms for Hadamard Matrices

Jennifer Seberry

We are finding that all the "best" algorithms are giving no solutions for orders that can only now be computed.

New Uses for Old Structures: Using BIBDs and Latin Squares to Construct Choice Experiments

Deborah Street and Leonie Burgess

People make choices all the time - just think of any visit to the supermarket or to a restaurant. Experiments which show people a number of options, be they meals, computers or medical treatments, and ask them to choose the option they think is best, are called *choice experiments*.

Choice experiments are used by marketers to predict the likely market share of new products, by governments to predict the likely response of the electorate to new policies and by health professionals to investigate prescribing patterns, amongst other applications. A well-designed choice experiment provides a cost-effective way to investigate these issues.

In most choice experiments, the options to be considered are described by a number of *attributes* (sometimes called *factors*), and each attribute has a number of possible *levels*. Each option is described by a level of each of the attributes. Each choice set has a fixed number of options in it, typically between 2 and 6, and each respondent will be shown a number of choice sets in turn and asked to choose their preferred option from each of the choice sets.

The mathematical problem is how best to arrange the options into choice sets so that good predictions can be made using a reasonable number of choice sets (so that respondents do not get too tired).

In this talk we show how BIBDs and Latin squares can be used to design choice experiments which have both good statistical efficiency and good respondent efficiency.