Graph decompositions and related trade structures

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Abstracts of invited and contributed talks. This file was updated on 25 July 2007.

Change Detection via Spectral Analysis in Dynamic Enterprise Networks

Diane Donovan
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In the present global environment, enterprise communication networks are continually expanding, both in terms of size and complexity. Consequently, an important aspect of network management is the development of efficient tools that ensure robust network performance, and monitor changes in both network topology and complexity. We are specifically interested in detecting changes which are the result of abnormal events or trends.

By representing networks as combinatorial graphs we gain access to a rich mathematical environment that facilitates a rigorous study the dynamical aspects of these structures.

In this talk we will compare different distance measures which have been developed to detect changes in network traffic. In particular, recent results using spectral analysis will be reported. Limitations of this approach will be discussed and problems for future investigation will be presented.

Graphs as Linked Structures

Roger B. Eggleton
Illinois State University

Our general context is the study of graphs with isomorphic induced subgraphs. Let G be a given graph and let K be an induced proper subgraph of G. A *linked structure* with *initial link* G and *kernel* K is a graph H with a proper automorphism $\pi : H \to H$ such that

(1) G is an induced subgraph of H;

(2) H is the union of the links $\pi^i(G)$, $i \in \mathbb{Z}$;

(3) the intersection of any two consecutive links $\pi^i(G) \cap \pi^{i+1}(G)$, is isomorphic to K.

Most graphs with a nontrivial automorphism turn out to be linked structures, so this viewpoint helps to clarify the structure of such graphs.

Furthermore, linked structures provide a tool for understanding *pseudo-similar subgraphs*. These generalize *pseudo-similar vertices*, which are vertices u, v of a graph G such that the induced subgraphs $G - u$ and $G - v$ are isomorphic, yet no automorphism of G maps u to v.

The ideas and results to be reported arise from current joint work with Peter Adams and Jim MacDougall.

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Domination in graphs

Abdollah Khodkar
Department of Mathematics, University of West Georgia

In a graph G, a vertex dominates itself and its neighbors. A subset S of $V(G)$, the vertex set of G, is a dominating set of G if S dominates every vertex of G at least once. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. A subset $S \subseteq V(G)$ is a double dominating set of G if S dominates every vertex of G at least twice. The double domination number $dd(G)$ is the minimum cardinality of a double dominating set of G. In this talk we first present a brief history of domination in graphs. Then we study domination number of grid graphs. If we have time we will see some results on perfect (exact) double domination number of gird graphs.

Keywords: dominating set, domination number; double domination number

Estimating the Number of s-t Paths in a Graph

Dirk Kroese
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The problem of counting the number of s-t paths in a graph is #P-complete. We provide an algorithm to estimate the solution stochastically, using sequential importance sampling. We show that the method works effectively for both graphs and digraphs. We also use the method to investigate

the expected number of s-t paths in a random graph of size n and density d, and develop a model that shows how this quantity behaves when n and d are varied.

A constraint on the biembedding of Latin squares

James Lefevre
The University of Queensland

A biembedding of two latin squares of order n is equivalent to a face 2-colourable triangulation of $K_{n,n,n}$. We consider the following question: Given two latin squares A and B of order n, is there any relabelling of A and B for which there exists a biembedding?

Grannell, Griggs and Knor answered this question computationally for $n \leq 7$. A *main class* of latin squares is a set of latin squares which are equivalent under some relabelling. The number of main classes and distinct biembeddings increases rapidly with n, so that there is relatively little information for $n \leq 6$, while the problem is not computationally feasible for $n \geq 8$. For $n = 7$ a pattern emerged without parallel in the equivalent results for Steiner triple systems; the 147 main classes partition into 16 sets, such that a biembedding exists for most pairs of main classes from the same set, but there is no biembedding between any two latin squares which are not in the same set.

Using a argument based on permutation parity, we give a necessary condition explaining this pattern, and briefly explore some other implications of this result.

Based on joint work with Diane Donovan, Mike Grannell and Terry Griggs.

Embeddings and Intersections

Chris Rodger
Auburn University

Trades have been a useful tool in obtaining good (but not best) embedding results for partial graph decompositions, and for finding two graph decompositions that have a prescribed intersection. In this talk, the technique will be used to find small embeddings for partial 4-cycle systems of arbitrary index, thus bringing latest results in line with the size for multiplicity one. Steiner triple systems where the intersection is required to be a partial parallel class will also be discussed.

Some results on 2-perfect cube decompositions of K_v and $K_v - F$

Mary Waterhouse
The University of Queensland

A 3-cube decomposition $\mathcal{Q}$ of a graph G is said to be 2-perfect if for every edge $\{x, y\} \in E(G)$, x and y are connected by a path of length 1 in exactly one cube of $\mathcal{Q}$, *and* x and y are connected by a path of length 2 in exactly one cube of $\mathcal{Q}$.

In this presentation we will consider a construction method which makes use of nested K_3-decompositions, thus settling the existence question for decompositions of $K_v - F$, where $v \equiv 2 \pmod{12}$. We will also briefly discuss some other (non-)existence results relating to decompositions of K_v.

This work has been done in collaboration with Dr James Lefevre and Prof. Peter Adams.

A new algorithm for finding the full set of minimal defining sets of t-designs

Emine Şule Yazıcı
Koç University, İstanbul/Turkey

A defining set of a t-(v,k,λ_t) design is a set of blocks which is a subset of a unique t-design with the given parameters. A minimal defining set is a defining set, none of whose proper subsets is a defining set. A smallest defining set is one with smallest cardinality. This paper proposes a new and more efficient algorithm that finds all non-isomorphic minimal defining sets of a given t-design. The complete list of minimal defining sets of the full 2-(7,3,5) design, 2-(15,3,1) designs, 2-(25,5,1) design and 2-(31,6,1) design were given to illustrate the efficiency of the algorithm.