Designs, Graphs and their Topological Representations

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The University of Queensland

Abstracts of invited and contributed talks. This file was updated on 5 April 2007.

How to construct a random design

Nicholas J. Cavenagh
UNSW

The word "random" is often associated with probability theory and analysis. However there are methods to generate and analyse random combinatorial designs using only discrete mathematics. This talk gives an introduction to these methods, with a focus on latin squares.

Indivisible half plexes

Judith Egan and Ian M. Wanless
Monash University

A k-plex is a selection of kn entries of a latin square of order n in which each row, column and symbol is represented precisely k times. A transversal of a latin square corresponds to the case $k = 1$. We show that for all even $n > 2$ there exists a latin square of order n composed of two parallel $\frac{1}{2}n$-plexes neither of which contains a k-plex for any value of k in the interval $0 < k < \frac{1}{2}n$.

Consecutively row Hamiltonian Latin squares and surface embeddings of K_n and $K_{n,n,n}$

Mike Grannell*
The Open University

Face 2-colourable triangular surface embeddings of $K_{n,n,n}$ correspond to biembeddings of Latin squares whose (row, column, entry) triples determine the faces. Biembeddings of Latin squares can be used in recursive constructions for face 2-colourable triangular surface embeddings of K_n, and these correspond to biembeddings of Steiner triple systems whose blocks determine the faces. A fundamental question is whether or not every Steiner triple system has a surface biembedding. A positive answer to this would imply that there are at least $n^{n^2(\frac{1}{6}-o(1))}$ nonisomorphic face 2-colourable triangular surface embeddings of K_n. How close can we get to this bound? A Latin square$A = (a_{i,j})$ of side n is *consecutively row Hamiltonian* if, for each i, $1 \leq i \leq n$, the permutation

$$\begin{pmatrix} a_{i,1} & a_{i,2} & \dots & a_{i,n} \\ a_{i+1,1} & a_{i+1,2} & \dots & a_{i+1,n} \end{pmatrix}$$

is a cycle of length n. Any such square has a biembedding with a copy of itself. What can we say about such squares? How many Latin squares have this property? Some partial answers to the foregoing questions will be provided.

* This is joint work with Terry Griggs, and it also draws on work with Geoff Bennett, Paul Bonnington, Martin Knor, Vladimir Korzhik and Jozef Širáň.

$(6,3,2)$s by the dozen

Ken Gray
University of Queensland

This talk shows how you can make a pocket catalogue of the blocks of all twelve $(6,3,2)$ BIBDs, using regular solids, so that your catalogue will survive going through the wash.

Rev. T. P. Kirkman and his work on Combinatorial Design Theory

Terry S. Griggs
The Open University

For most of the second half of the nineteenth century, Rev. Thomas Pennington Kirkman (1806 - 1895) was Rector of Croft, a small country parish in Lancashire, England. He would have been an obscure, anonymous clergyman except that he published over 70 mathematical papers on a variety of topics including Designs, Partitions, Groups, Polyedra (sic) and Knots. He held no academic appointment but became a Fellow of the Royal Society. He is most widely known for the "Kirkman schoolgirls problem" but there is much more to his work on Combinatorial Design Theory. He wrote the first papers on the subject and introduced ideas which are still current. As all design-theorists know he proved the existence of Steiner triple systems for all admissible orders six years before Steiner asked the question. In this talk I give an outline of the life of this remarkable Englishman, whose first paper was published when he was over 40 and last just before his death, and discuss his work on combinatorial designs.

Latin bitrades and tessellations

Carlo Hämäläinen
University of Queensland

A latin bitrade $(T^{\diamond}, T^{\otimes})$ can be represented, up to isotopism, by three permutations τ_1, τ_2, and τ_3 acting on a finite set Ω, where $\tau_1\tau_2\tau_3 = 1$. In this talk I will explain how this permutation representation induces a bipartite graph Γ embedded in a surface S. By considering the *universal covering* of S it is possible to deduce combinatorial information about the underlying graph Γ and then about the original latin bitrade. The universal covering also allows us to see the intrinsic spherical, Euclidean, or hyperbolic tessellations induced by latin bitrades.

$L(p, q)$-Labelling of outerplanar graphs

Michael Hoffmann
University of Leicester

Labelling or Colouring of graphs is a wide and well studied area. We introduce a technique to compose graphs while maintaining enough information about the labelling. This technique will be demonstrate on the $L(p, q)$ labelling on outerplanar graphs. The results obtained do improve previously known lower and upper bounds.

In details: An $L(p, q)$-labelling of a graph G is a labelling of the vertices by the integers, where the labels of adjacent vertices differ by p and the labels of vertices at distance two differ by q. We will present an algorithm that decides whether every outerplanar graph of degree at most Δ can be $L(p, q)$-labelled with n labels. We apply this algorithm to three different labelling problems for outerplanar graphs. We give precise bounds for $L(2,1)$-labelling outerplanar graphs where $\Delta = 3$ and increase the previously known lower bounds when $\Delta = 4, 5$ and 6. We give precise bounds for $L(1,1)$-labelling outerplanar graphs where $\Delta = 4, 5$. We give a precise bound when $L(0,1)$-labelling outerplanar graphs where $\Delta = 3$.

In practice precise bounds can be found for small degree Δ. The challenge to understand the (possible large) counter examples remains. Their structure could give rise to improved bounds for any degree.

Cycle Decompositions of Complete Graphs

Daniel Horsley

If we wish to decompose a complete graph of order n into edge-disjoint cycles of lengths $m_1, m_2, \ldots, m_t$ then the following obvious necessary conditions must hold.

- $3 \leq m_1, m_2, \ldots, m_t \leq n$.
- n is odd.
- $m_1 + m_2 + \cdots + m_t = \binom{n}{2}$.

In 1981 Alspach posed the problem of showing that these necessary conditions were also sufficient. In this talk we will discuss various existing results on Alspach's problem as well as introduce a new result. In particular we will examine what "fraction" of the problem these results solve.

Cycle Decompositions of Circulant Graphs

Geoff Martin
University of Queensland

To decompose a graph into cycles of lengths $m_1, m_2, \ldots, m_t$, there are a number of well-known obvious necessary conditions. It would be of great interest if we could find an infinite family of graphs for which these necessary conditions are also sufficient. Alspach conjectured in 1981 that this is the case for the complete graph with an odd number of vertices and the complete graph minus a one factor with an even number of vertices. We consider the problem of whether this is also the case for the circulant graph on n vertices with connection set $\{1, 2, \ldots, k\}$ where $3 \le k \le \lfloor \frac{n-1}{2} \rfloor$. We will discuss some preliminary results.

Divisors of the number of Latin rectangles

Douglas Stones and Ian M. Wanless
Monash University

A $k \times n$ *Latin rectangle* is a $k \times n$ array with entries from $\{1, 2, \ldots, n\}$ such that every row and every column contains distinct symbols. Let $R_{k,n}$ denote the number of $k \times n$ Latin rectangles with first row $1, 2, \ldots, n$ and first column $1, 2, \ldots, k$. $R_{k,n}$ displays several curious divisibility properties. For example: For prime n, $R_{k,n} \equiv 1 \pmod{n}$ and for composite n, $R_{k,n} \equiv 0 \pmod{n}$ if and only if k is greater than the largest prime divisor of n.

An enumeration of small latin trades

Ian Wanless
Monash University

Suppose I have two latin squares L and M of the same order. The entries of L which differ from the corresponding entry in M form what is known as a latin trade, say T. The size of T is the number of entries in it. If we take T together with the corresponding entries in M this forms what is known as a latin bi-trade.

In this talk I'll discuss a computer enumeration of latin trades and bi-trades of sizes up to 19. The (bi-)trades were counted according to various equivalences and those with special properties (such as being minimal or homogeneous) were identified. I'll also look at minimal embeddings of the trades in latin squares and of the bi-trades in topological surfaces. The catalogue has already resulted in some open questions being answered and in the discovery by Cavenagh and Lisonek of a new connection between spherical bi-trades and 3-connected cubic bipartite planar graphs.

Non-Sequential Arrays with Sequential Shadows

Marks R Nester[1] and Anne Penfold Street[2]
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A *finite sequence* of length n is written $s_1, s_2, \ldots, s_n$. A *periodic sequence* of period n is written $\ldots, s_n, s_1, s_2, \ldots, s_n, s_1, s_2, \ldots, s_n, \ldots$, though it may use different starting points.

A *sequential array* is one where each row and each column is occupied by the same periodic sequence, which may occur in reverse order. Thus the periodic quaternary sequence $\ldots, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, \ldots$ gives rise to two sequential arrays, namely,

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{array} \quad \text{and} \quad \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{array}$$

which have corresponding labels $[1, -2, 3, -4]$ and $[1, 4, 3, 2]$ respectively. The minus signs indicate that the sequence occurs in reverse order in columns 2 and 4 of the first array.

Sequential arrays were originally studied in response to a question asked by R M Cormack in relation to the layout of agricultural experiments; we do not know whether he was the original source of the problem.

Assume that we are working over the alphabet $A_q = \{1, 2, \ldots, q\}$, that the sequence S contains all the characters of A_q and that $Y = \{c_1, c_2, \ldots, c_r\}$ is a nonempty subset of A_q. The *shadow sequence* $\sigma_{c_1,c_2,\ldots,c_q}$ of S with respect to Y is

$$[\sigma_{c_1,c_2,\ldots,c_q}]_i = \begin{cases} 1 & \text{if } [S]_i \in Y, \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the *component sequence* $\kappa_{c_1,c_2,\ldots,c_q}$ of S with respect to Y is

$$[\kappa_{c_1,c_2,\ldots,c_q}]_i = \begin{cases} [S]_i & \text{if } [S]_i \in Y, \\ 0 & \text{otherwise.} \end{cases}$$

For example, if $S = (1, 2, 1, 1, 2, 3, 4)$ then some of its shadows are $\sigma_1 = (1, 0, 1, 1, 0, 0, 0)$, $\sigma_2 = (0, 1, 0, 0, 1, 0, 0)$ and $\sigma_{1,2} = (1, 1, 1, 1, 1, 0, 0)$. The corresponding components are $\kappa_1 = (1, 0, 1, 1, 0, 0, 0)$, $\kappa_2 = (0, 2, 0, 0, 2, 0, 0)$ and $\kappa_{1,2} = (1, 2, 1, 1, 2, 0, 0)$.

If M is a sequential array derived from sequence S using labels L, then we show that every possible shadow or component of M is also a sequential array based on L. For an array to be sequential, it seems plausible that it is sufficient for all possible nontrivial shadows to form sequential arrays based on L, but this is not the case. We consider some counterexamples.