

MATH 3403
TUTORIAL SHEET 8

1. Solve

$$u_{xx} - u_{tt} = x^2; \quad 0 < x < 1; \quad 0 < t < \infty$$

where

$$\begin{aligned} u(x, 0) &= 0; \\ u_t(x, 0) &= 0; \\ u(0, t) &= 0; \\ u_x(1, t) &= 0. \end{aligned}$$

(Hint: Find a particular solution of the equation as a function of x which satisfies the boundary conditions.)

2. Find the solution of Laplace's equation ($u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$) in the wedge $0 \leq r < 1, -\frac{\pi}{3} < \theta < \frac{\pi}{3}$ with the boundary conditions $u(r, \frac{\pi}{3}) = u(r, -\frac{\pi}{3}) = 0, u(1, \theta) = \frac{\pi}{3} - |\theta|$.

3. Find the solution of Laplace's equation ($u_{xx} + u_{yy} = 0$) for $0 < x < 1, 0 < y < 1$ with the boundary conditions $u(1, x) = x - \frac{1}{2}x^2, u(0, x) = 0, u(y, 0) = 0, u_x(y, 1) = 0$.