

MATH 3403
TUTORIAL SHEET 6

1. Find a weight factor which will convert the following operators to self adjoint form.

(a) Tchebychev's operator

$$\mathcal{T}(y) = (1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx}$$

(b) Laguerre's operator

$$\mathcal{L}(y) = x \frac{d^2y}{dx^2} + (1 - x) \frac{dy}{dx}$$

(c) Hermite's operator

$$\mathcal{H}(y) = \frac{d^2y}{dx^2} - 2x \frac{dy}{dx}$$

(d) The cylindrical Laplace operator

$$\mathcal{C}(y) = \frac{d^2y}{dr^2} + \frac{1}{r} \frac{dy}{dr}$$

2. The Tchebychev operator, (1(a)), has eigenfunctions $T_n(x)$ on the interval $[-1, 1]$ which are polynomials of degree n in x ;

$$T_n(x) = \sum_{r=0}^n a_r x^r ; n = 0, 1, 2, \dots$$

scaled so that $T_n(1) = 1$.

Determine the first four eigenfunctions and the corresponding eigenvalues.

3. Solve the following Euler equations for $x > 0$.

(a) $x^2 y'' + 2xy' - 2y = 0$

(b) $x^2 y'' - xy' - 3y = 0$

(c) $2x^2 y'' + xy' - y = 0$

(d) $x^2 y'' + xy + y = 0$

Assignment Questions 1(a) and 2.