

MATH 3403
TUTORIAL SHEET 5

1. Determine the eigenvalues and eigenfunctions for the following problems.

(a) $X'' + \lambda X = 0 ; X'(0) = X'(1) = 0$

((b)) $X'' + \lambda X = 0 ; X(0) = 0 , X'(1) = X(1)$

(c) $x^2 X'' + xX' + \lambda X = 0 ; X(1) = X(2) = 0$

2. Verify that the functions $1, \{\cos(nx)\}$ $n = 1, 2, \dots$, are orthogonal on the interval $[0, \pi]$.

Expand the function $f(x) = \sinh x$, $0 \leq x \leq \pi$, in the series form

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) .$$

3. Find the solution of the finite wave equation

$$u_{xx} = u_{tt} ; 0 \leq x \leq 1 ; t \geq 0$$

which satisfies the initial conditions

$$u(x, 0) = 2x - x^2 ; u_t(x, 0) = 0$$

and the boundary conditions

$$u(0, t) = 0 ; u_x(1, t) = 0 .$$

Assignment Question 3.