MATH 3403

TUTORIAL SHEET 4

1. Find the solution to

$$u_{xx} = 4u_{tt}, \quad 0 < t < \infty, \quad 0 < x < \infty$$

with initial conditions

$$u(x,0) = \sin x, \quad x > 0$$
$$u_t(x,0) = -\cos x, \quad x > 0$$

and boundary condition

$$u_x(0,t) = 0, \quad t > 0.$$

2. Find $u(x,t)$ for $t > 0$, $-\infty < x < \infty$, if

$$\begin{cases} u_{xx} = u_{tt} \ x < 0 \\ u_{xx} = 4u_{tt} \ x > 0 \end{cases}$$
$$\begin{cases} u(x,0) = \sin x \ x < 0 \\ u(x,0) = 0 \ x > 0 \end{cases}$$
$$u_t(x,0) = 0$$

3. Solve the moving boundary problem

$$u_{xx} - u_{tt} = 0 \ ; \quad 0 < t < \infty \ , \ \frac{1}{2}t < x < \infty$$
$$u(x,0) = e^{-x} \ , \ u_t(x,0) = e^{-2x} \ ; \quad 0 < x < \infty$$
$$u\left(\frac{1}{2}t, t\right) = e^{-t} \ ; \quad 0 < t < \infty$$

4. Find particular solutions of the following inhomogeneous wave equations.

(a) $$u_{xx} - c^{-2}u_{tt} = \cos x$$

(b) $$u_{xx} - c^{-2}u_{tt} = \sinh t$$

(c) $$u_{xx} - c^{-2}u_{tt} = x^2 + t^2$$

(d) $$u_{xx} - c^{-2}u_{tt} = (x+t)^2$$

5. Find the solution of the initial/boundary value problem

$$u_{xx} - \frac{1}{4}u_{tt} = \sin(x+t) \ ; \quad 0 < x < \infty \ ; \ 0 < t < \infty$$
$$u(x,0) = \cos x \ ; \ u_t(x,0) = \sin x \ ; \quad 0 < x < \infty$$
$$u_x(0,t) = 0 \ ; \quad 0 < t < \infty \ .$$

6. Find the solution of the telegraph equation

$$u_{xx} = u_{tt} + 4u_t + 4u$$

with the initial conditions

$$u(x,0) = e^x \ ; \ u_t(x,0) = e^x \ .$$

Assignment Questions 2 and 5.