

MA311
ASSIGNMENT 4
2000

WHO WANTS TO BE A MILLIONAIRE?

Imagine that you have some initial capital which you hope to increase by investing in the futures market on a week-by-week basis.

The rate of return which you can expect will depend on the amount of information which you have at any time, and some capital will have to be expended each week in order to obtain this information, since it rapidly becomes obsolete. The proportion of your available capital which you expend on acquiring information will be the control variable for the problem. Assume that you know nothing to begin with.

Returns from investment each week are assumed to be given by

$$P = (f(k) - 1)I$$

where

P is the profit;

I is the amount invested;

k is the 'knowledge level' at the beginning of the week;

and

f is a function such that

$$f(0) = 1 ; f'(k) > 0 ; f''(k) < 0 .$$

The knowledge level itself is assumed to satisfy the relationship

$$k(n + 1) = rk(n) + g(e)$$

where

$k(n)$ is the 'knowledge level' at the start of week n ;

r ; $0 < r < 1$ represents the amount of information which remains useful;

e is the amount in thousands of dollars spent on acquiring information;

and

g is a function such that

$$g(0) = 0 ; g'(e) > 0 ; g''(e) < 0 .$$

1. Set up a set of controlled difference equations describing this system.
2. Show that there is a threshold amount of initial capital below which it is not possible to make money.
3. Given that

$$f(k) = \frac{1 + 1.2k}{1 + k}$$
$$g(e) = \log(1 + e)$$
$$r = 0.5$$

and the initial capital is \$5000 ,

(a) find the constant proportion of capital, which, if spent on acquiring information, produces the greatest profit at the end of 50 weeks.

(b) Use this suboptimal solution as a starting point to determine the optimal investment strategy to maximise the profit over the same period.