

**MA311**  
ASSIGNMENT 3  
2000

BIKE RIDING

A cyclist starts from rest and rides 1 kilometre along an undulating track, reaching the end  $T$  seconds later. The elevation,  $h$  metres, of the track is represented as a function  $h(s)$  of the distance  $s$  metres along the track from the starting point.

Her progress is governed by the equation

$$\frac{dv}{dt} = u - \alpha v^2 - g \frac{dh}{ds} ,$$

where

$v = \frac{ds}{dt}$  is her speed;

$u$  ( $0 \leq u \leq 3$ ) is her applied effort;

$-\alpha v^2$  is the drag;

and  $-g \frac{dh}{ds}$  is the gravitational acceleration.

Given that  $g = 10$  and  $\alpha = .01$ , determine the function  $u$  which minimises

$$\int_0^T (2 + u) dt$$

when

$$h(s) = \cos(\pi s/500) .$$

What is her average speed for this optimal solution?