

MA311

ASSIGNMENT 2 2000

OPTIMAL STUDYING

A typical student wishes to pass while doing as little work as possible.

The subject is assessed by an end of semester examination, and it is assumed that the mark he gains reflects perfectly his percentage knowledge K of the subject matter formally studied in the course.

The student initially has no knowledge of the subject matter; that is $K_1 = 0$.

For every W hours of study he puts in per week, the student increases K by an amount proportional to $W/(1+W)$. Unfortunately, this student also forgets a constant proportion c of the subject matter each week. Therefore, if K_n denotes his knowledge level at the start of week n , and W_n denotes the number of hours he spends studying in this week, we have

$$K_{n+1} = aK_n + b \frac{W_n}{1+W_n},$$

where $a = 1 - c$.

The examination occurs at the beginning of the sixteenth week.

Given that $b = 10$ and $c = 0.05$, determine the values of $\{W_1, \dots, W_{15}\}$ which minimise

$$J = \sum_{n=1}^{15} W_n,$$

while ensuring that

- (a) $K_{16} = 50$; the student achieves a grade of 4.
- (b) $K_{16} = 65$; the student achieves a grade of 5.
- (c) $K_{16} = 75$; the student achieves a grade of 6.
- (d) $K_{16} = 85$; the student achieves a grade of 7.

Calculate J explicitly for case (d), and compare this result with the amount of time spent to achieve the same final value of K by choosing $W_n = \text{constant}$.

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