

**MP213**  
LAPLACE TRANSFORMS  
SELF-HELP EXERCISES USING MAPLE

These notes are intended to guide students in the use of the Laplace transform capabilities of **Maple**.

**Maple** is a symbolic manipulation package for doing mathematics. It can be accessed on the PCs in the laboratories in rooms 4.20 and 5.19 by calling up “Applications” and then clicking the mouse on the appropriate icon.

If you are unfamiliar with **Maple**, see me for some basic notes on how to use the system. There are certain conventions which must be observed.

Since the routines used in Laplace transform manipulations are not part of the core set of instructions, the first instruction you should type is

**readlib(laplace):**

(Note: the colon ‘:’ is important. This is one of the conventions mentioned above.)

The two principal commands which are used are **laplace** and **invlaplace**.

The instruction

**laplace**(*funct*(*t*), *t* , *s*);<sup>1</sup>

calculates the Laplace transform of the function specified by *funct*(*t*) .

**Exercises 1.**

Use **Maple** to find the Laplace transforms of

(a)  $\exp(a * t)$  ; (b)  $\exp(-t) * \sin(2 * t)$  ; (c)  $t^2 * \exp(-t)$  .<sup>2</sup>

The instruction

**invlaplace**(*Funct*(*s*), *s* , *t*);

calculates the inverse Laplace transform of the function specified by *Funct*(*s*).

**Exercises 2.**

Use **Maple** to find the inverse Laplace transforms of

(a)  $1/(s + 2)$  ; (b)  $(s + 2)/(s^2 + 1)$  ; (c)  $s^2/(s + 1)^3$  .

If you do not specify the function *funct*(*t*), but leave it as *y*(*t*), **Maple** will carry the expression *laplace*(*y*(*t*),*t*,*s*) as a symbolic variable.

**Exercises 3.**

Use **Maple** to find

(a) **laplace**(**diff**(*y*(*t*),*t*),*t*,*s*); ;

(b) **laplace**(**int**( $\exp(\mathbf{t-u}) * \mathbf{y(u)}$ ,  $\mathbf{u = 0.. t}$ ),*t*,*s*); .

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<sup>1</sup>Maple requires the semi-colon ‘;’ at the end of each executable instruction

<sup>2</sup>Powers  $t^n$  are entered in maple as **t^n**.

## SOLVING DIFFERENTIAL EQUATIONS

Consider the following initial value problem:

$$\ddot{y} + 2\dot{y} - 3y = e^t ; y(0) = 1 ; \dot{y}(0) = -1 .$$

In order to see the nature of the **Maple** commands which can be used to find the solution by means of the Laplace transform, carry out the following sequence of instructions, and enter the results in the spaces provided.

**eq:= diff(y(t),t,t) + 2\*diff(y(t),t) - 3\*y(t) = exp(t);**

*This enters the differential equation, and gives it the label eq*

**leq:= laplace(eq,t,s);**

*This takes the Laplace transform of the equation. The transformed equation is labelled leq*

**sub:= subs(y(0)=1,D(y)(0)=-1,leq);**

*This substitutes the given initial values into the transformed equation.*

**Y:= solve(sub,laplace(y(t),t,s));**

*This solves the transformed equation for  $\mathcal{L}(y)$  as a function of  $s$ .*

**Y1:= normal(Y);**

*This tidies up the algebra*

**Y2:= convert(Y1,parfrac,s);**

*This determines the partial fraction expansion of Y1*

**yp:= invlaplace(Y2,s,t);**

*Finally, using the inverse Laplace transform to obtain the solution.*

We can use the computer to check that this is indeed the solution by calculating;

**diff(yp,t,t) + 2\*diff(yp,t) - 3\*yp;**

**subs(t=0,yp);**

**dyp:= diff(yp,t);**

**subs(t=0,dyp);**

Note: The steps **Y1** and **Y2** have been included to assist students when checking their own calculations. **Maple** can go straight to **yp := invlaplace(Y,s,t);**.

#### **Exercises 4.**

Use the Laplace transform to solve the following initial value problems.

Use **Maple** to check your calculations at each stage.

(a)  $\ddot{y} - 3\dot{y} + 2y = t^2 + t + 1 \quad y(0) = 1, \dot{y}(0) = 2 .$

(b)  $\ddot{y} - 2\dot{y} + 5y = \cos(2t) \quad y(0) = 0, \dot{y}(0) = 0 .$

(c)  $\ddot{y} + 2\dot{y} + y = e^{-t} \quad y(0) = 1, \dot{y}(0) = 0 .$

## CONVOLUTIONS

In exercise 3(b), Maple found the Laplace transform of an integral of the form

$$h(t) = \int_0^t f(t-u)g(u) du ,$$

which is known as the *convolution* of the functions  $f$  and  $g$ . The convolution has the important property

$$\mathcal{L}(h) = \mathcal{L}(f) \cdot \mathcal{L}(g)$$

where  $\mathcal{L}$  represents the Laplace transform.

**Exercises 5.**

Use the sequence of instructions

**F:= laplace(f(t),t,s);**

**G:= laplace(g(t),t,s);**

**h:= invlaplace(F\*G,s,t);**

to evaluate  $\int_0^t f(t-u)g(u) du$  in the cases

(a)  $f(t) = \exp(t)$  ;  $g(t) = \sin(t)$  :

(b)  $f(t) = t^2$  ;  $g(t) = \cos(2 * t)$  :

(c)  $f(t) = \sin(a * t)$  ;  $g(t) = \cos(b * t)$  :

and check the results via

**int(f(t-u) \* g(u), u = 0.. t);**

Use **Maple** to solve the following integral equations.

(a) 
$$y(t) = 1 + \int_0^t \sin(t-u)y(u) du$$

(b) 
$$\int_0^t \frac{y(u)}{\sqrt{t-u}} du = 1$$