

MATH 2000
TUTORIAL SHEET 1

1. Let (x, y, z) denote a typical element in \mathbb{R}^3 .
Which of the following subsets of \mathbb{R}^3 are subspaces?

- (i) The set for which $x - y + z = 0$.
- (ii) The set for which $x - y + z = 1$.
- (iii) The set for which $x^2 = y^2$.
- (iv) The set for which $x^2 + z^2 = 0$.

2. Let V be the subspace of \mathbb{R}^3 spanned by

$$\{(1, 1, 1), (1, -1, 1)\}.$$

Which of the following vectors lie in V ?

- (a) $(1, 0, 0)$
- (b) $(0, 1, 0)$
- (c) $(1, 2, 3)$
- (d) $(1, 2, 1)$

3. Let V be the subspace of \mathbb{R}^4 spanned by

$$\{(0, 1, 3, -2), (1, 2, 6, 0), (2, 3, 9, 2), (1, 1, 3, 2)\}.$$

Find a basis for V , and express the vectors in the spanning set as linear combinations of the basis vectors.

4. Find the row reduced echelon forms for the following matrices.

(a)
$$\begin{pmatrix} 0 & 0 & 6 & 2 & -4 & -8 & 8 \\ 0 & 0 & 3 & 1 & -2 & -4 & 4 \\ 2 & -3 & 1 & 4 & -7 & 1 & 2 \\ 6 & -9 & 0 & 11 & -19 & 3 & 1 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 2 & 2 & -2 & 5 \\ 7 & 7 & 1 & 10 \\ 5 & 5 & -1 & 5 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1 & 1+i & -i \\ 0 & i & 1+2i \\ 1 & 1+2i & 1+i \end{pmatrix}$$

In each case, express the rows of the original matrix as linear combinations of the rows of the rre form.

5. Let A be the matrix

$$\begin{pmatrix} 1 & 2 & 4 & 4 \\ 1 & 1 & 3 & 2 \\ 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 2 \end{pmatrix}.$$

Find bases for the null space, row space and column space of A .

Show that the basis vectors of the row space and the null space together span \mathbb{R}^4 .

6. Let T be the linear transformation from the vector space V of all real polynomials in x of degree ≤ 3 , to the space W of polynomials of degree ≤ 2 given by

$$T(p(x)) = x \frac{d^2 p}{dx^2} - \frac{dp}{dx}.$$

Determine the matrix of the transformation relative to the bases $\{1, x, x^2, x^3\}$ for V and $\{1, x, x^2\}$ for W .

Find bases for the kernel and range of the transformation.

Assignment Question 4.