

$$\begin{aligned}
& \mathcal{L}^{-1}\left(\frac{1}{p-a}\right) \\
&= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{e^{px}}{p-a} dp, \quad c > \operatorname{Re}(a) \\
&= \text{residue at } p = a \text{ for } x > 0 \\
&= e^{ax}
\end{aligned}$$

$$\begin{aligned}
& \mathcal{L}^{-1}\left(\frac{1}{(p+1)(p-2)^2}\right) \\
&= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{e^{px}}{(p+1)(p-2)^2} dp, \quad c > 2 \\
&= \sum \text{residues at } p = -1 \text{ and } p = 2 \text{ for } x > 0
\end{aligned}$$

At  $p = -1$ , the residue is given by

$$\lim_{p \rightarrow -1} \frac{e^{px}}{(p-2)^2} = \frac{1}{9} e^{-x}$$

At  $p = 2$ , the residue is given by

$$\begin{aligned}
& \lim_{p \rightarrow 2} \frac{d}{dp} \left( \frac{e^{px}}{p+1} \right) \\
&= \lim_{p \rightarrow 2} \left( \frac{x e^{px}}{p+1} - \frac{e^{px}}{(p+1)^2} \right) \\
&= \frac{1}{3} x e^{2x} - \frac{1}{9} e^{2x}
\end{aligned}$$

$$\mathcal{L}^{-1}\left(\frac{1}{(p+1)(p-2)^2}\right) = \frac{1}{9}(e^{-x} + (3x-1)e^{2x})$$

$$\begin{aligned}
& \mathcal{L}^{-1}\left(\frac{p}{(p+1)^3(p-1)^2}\right) \\
&= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{p e^{px}}{(p+1)^3(p-1)^2} dp, \quad c > 1 \\
&= \sum \text{residues at } p = 1 \text{ and } p = -1 \text{ for } x > 0.
\end{aligned}$$

At  $p = 1$ , the residue is given by

$$\begin{aligned}
& \lim_{p \rightarrow 1} \frac{d}{dp} \left( \frac{p e^{px}}{(p+1)^3} \right) \\
&= \lim_{p \rightarrow 1} \left( \frac{e^{px} + x p e^{px}}{(p+1)^3} - \frac{3 p e^{px}}{(p+1)^4} \right) \\
&= \frac{1}{8}(x+1)e^x - \frac{3}{16}e^x = \frac{1}{16}(2x-1)e^x
\end{aligned}$$

At  $p = -1$ , the residue is given by

$$\begin{aligned}
& \frac{1}{2} \lim_{p \rightarrow -1} \frac{d^2}{dp^2} \left( \frac{pe^{px}}{(p-1)^2} \right) \\
&= \frac{1}{2} \lim_{p \rightarrow -1} \frac{d}{dp} \left( \frac{e^{px} + px e^{px}}{(p-1)^2} - \frac{2pe^{px}}{(p-1)^3} \right) \\
&= \frac{1}{2} \left( \frac{2xe^{px} + px^2 e^{px}}{(p-1)^2} - \frac{4(1+px)e^{px}}{(p-1)^3} + \frac{6pe^{px}}{(p-1)^4} \right) \\
&= \frac{1}{8}(2x - x^2)e^{-x} + \frac{1}{4}(1-x)e^{-x} - \frac{3}{16}e^{-x} \\
&= \frac{1}{16}(1 - 2x^2)e^{-x}
\end{aligned}$$

$$\mathcal{L}^{-1} \left( \frac{p}{(p+1)^3(p-1)^2} \right) = \frac{1}{16}(2x-1)e^x + \frac{1}{16}(1-2x^2)e^{-x}$$