2001 UQ/QAMT Competition Solutions, Year 9

1. Try drawing each later of the block, taking vertical slices. In the first layer, there are 3 missing, the second layer, there are 9 missing, in the third layer, there are 19 missing, in the fourth layer, there are 7 missing and in the (back) fifth layer, there are 3 missing, making a total of 43 missing leaving 125-43=82 blocks behind.

2. **Method 1** Let Alice's number be N. Then Bill's number is $10N+8$ and Carol's number is $1000 + N$. To win, we need Bill's number to be twice Carol's number, so

$$\begin{aligned} 10N + 8 &= 2(1000 + N) \\ 10N + 8 &= 2000 + 2N \\ 8N &= 1992 \\ N &= 1992/8 = 249 \end{aligned}$$

So the only winning number is 249.

Method 2 Trial and error. Start with a table:

Alice's no.	Bill's no.	Carol's no.	Twice Carol's no
100	1008	1100	2200
200	2008	1200	2400
300	3008	1300	2600
400	4008	1400	2800
500	5008	1500	3000
600	6008	1600	3200
700	7008	1700	3400
800	8008	1800	3600
900	9009	1900	3800

This shows that a winning number must be in the 200s. Indeed since Carol's number starts with a 1, twice Carol's number starts with a 2 or a 3. So Bill's number and hence Alice's number must start with a 2 or a 3. But if Alice's number starts with a 3, Carol's number is 13XX, so twice Carol's number starts with a 2 and so can't equal Bill's number.

Alice's no.	Bill's no.	Carol's no.	Twice Carol's no
210	2108	1210	2420
220	2208	1220	2440
230	2308	1230	2460
240	2408	1240	2480
250	2508	1250	2500
260	2608	1260	2520

This shows that a winning number must start with 24 or 25. Indeed since Carol's number starts with 12, twice for Carol's number starts with 24 or 25. If Alice's number if 25X then twie carol's number is $2\times125X = 250X$ or 251X. So only 250 or 251 are possible for a winning number. The table above shows 250 doesn't win and neither does 251 since $2518 \neq 2502 = 2 \times 1251$.

If Alice's number is 24X, then Carol's number is 124X so twice Carol's number is 248X or 249X, so only 248 or 249 are possible winning numbers. $2488 \neq 2496 = 2\times1248$, but $2498 = 2 \times 1249$, so 249, 2498 and 1249 are winning numbers.

3. There will be 17 pearls on each side of the middle pearl. Let the outside one on the \$100 side be worth \$a and the outsie one on the \$125 side be worth \$b. The values of the pearls on one side are:

$$a, a+100, a+200, \ldots, 1+1600, 1+1700(\text{middle pearl})$$

and on the other side

$$b, b+125, b+2\times 125, \ldots, b+16\times 125, b+17\times 125(\text{middle pearl})$$

Hence $a+1700 = b+17\times 125 = b+2125$, so $b = a-425$.

The total value of the 16 outside pearls on the \$100 side is

$$\begin{aligned} a+(a+100)+(a+200)+\ldots+(a+1600) \\ = 17a+100(1+2+\ldots+16) \\ = 17a+13600 \end{aligned}$$

Similarly, the value of the 16 outside pearlson the \$125 side is

$$\begin{aligned} b+(b+125)+(b+2\times 125)+\ldots+(b+16\times 125) \\ = 17b+125(1+2+\ldots+16) \\ = 17b+17000 \\ = 17(a-425)+17000 \\ = 17a+9775 \end{aligned}$$

Hence the total value of the string, including the middle pearl is

$$(17a+13600)+(17a+9775)+(a+1700) = 35a+25075$$

Thus $35a+25075 = 70575$ so $35a = 45500$ and hence $a = 1300$. Then $a+1700 = 1300+1700 = 3000$ so the middle pearl is worth \$3000.

4. From the angles in the triangle CDG, angle DHC + 45 + 90 = 180 so angle DHC = 45. From the angles at H, angle EHA + 90 +45 = 180, so angle EHA = 45. Similarly angle AEH = angle FEB = angle BFE = angle GFC = angle FCH = 45. Thus AEH, BFE, GFC and DHC are all isoscelese right angle triangles. So HA = AE = 3 cm and EH = $1.414\times$ AE = $1.414\times 3 = 4.242$ cm. Since EFGH is a rectange, FG=EH = 4.242. Then from triange GFC, FC = $1.414\times$ FG = $1.414\times 4.242 = 5.998 \approx 6$ cm. Since FC is three times the length of BF, BF = 2cm. Then EB = BF = 2cm, so AB = AE+EB = 3+2 = 5cm, and BC = BF+FC = 2+6 = 8cm. Thus, area ABCD = AB $\times$ BC = $5\times 8 = 40$ cm^2.

5.