2011 UQ/QAMT Problem Solving Competition - Year 11 & 12 Paper

All questions have equal value.

Question 1
How many integers are of the form $2^n$ (where $n$ is a natural number) such that after deleting the first digit of its decimal representation we again get a power of 2?

Question 2
The famous haberdasher’s problem involves cutting an equilateral triangle so that the pieces can be rearranged into a square. One solution is given in the following diagram:

Suppose the triangle has area 1. What is the length of the longest cut in the triangle?

Question 3
Let $a$, $b$ and $c$ be distinct non-zero real numbers such that

$$a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}.$$ 

What is the value of $|abc|$?

Question 4
Draw $n$ equally spaced marks between 0 and 1 on each of the $x$ and $y$ axes. Connect the first mark on the $x$-axis (closest to 0) to the last mark on the $y$ axis (closest to 1), the second mark on the $x$ axis to the second last on the $y$ axis, and so on.

As $n$ increases this process creates a curved boundary, $C$, as seen in this example diagram. Give an equation for this curve as $n \to \infty$.

Question 5
Suppose $r$ and $s$ are positive integers with no common factor and

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{1339} - \frac{1}{1340} = \frac{r}{s}.$$ 

Find a prime factor of $r$. 

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