

1. There will be 17 pearls on each side of the middle pearl. Let the outside one on the \$100 side be worth \$ a and the outside one on the \$125 side be worth \$ b . The values of the pearls on one side are:

$$a, a + 100, a + 200, \dots, 1 + 1600, 1 + 1700(\text{middle pearl})$$

and on the other side

$$b, b + 125, b + 2 \times 125, \dots, b + 16 \times 125, b + 17 \times 125(\text{middle pearl})$$

Hence $a + 1700 = b + 17 \times 125 = b + 2125$, so $b = a - 425$.

The total value of the 16 outside pearls on the \$100 side is

$$\begin{aligned} a + (a + 100) + (a + 200) + \dots + (a + 1600) \\ = 17a + 100(1 + 2 + \dots + 16) \\ = 17a + 13600 \end{aligned}$$

Similarly, the value of the 16 outside pearls on the \$125 side is

$$\begin{aligned} b + (b + 125) + (b + 2 \times 125) + \dots + (b + 16 \times 125) \\ = 17b + 125(1 + 2 + \dots + 16) \\ = 17b + 17000 \\ = 17(a - 425) + 17000 \\ = 17a + 9775 \end{aligned}$$

Hence the total value of the string, including the middle pearl is

$$(17a + 13600) + (17a + 9775) + (a + 1700) = 35a + 25075$$

Thus $35a + 25075 = 70575$ so $35a = 45500$ and hence $a = 1300$. Then $a + 1700 = 1300 + 1700 = 3000$ so the middle pearl is worth \$3000.

2. Let the total number of sheep on the farm be t . Then there were $t/3$ merinos, $2/7t$ suffolks and $t/5$ border leicesters. Now consider the sheep in the bottom paddock. Here there were $1/40$ of the merinos, so $1/40 \times t/3 = t/120$ merinos; $1/16$ of the suffolk, so $1/16 \times 2/7t = t/56$ suffolks and $1/9$ of the border leicesters, so $1/9 \times t/5 = t/45$ border leicesters. Hence, 120, 56, 45 must all be factors of t (for there is a whole number of each breed in the bottom paddock). Now $120 = 8 \times 3 \times 5$; $56 = 8 \times 7$, $45 = 9 \times 5$. Hence $8 \times 9 \times 5 \times 7$ must be a factor of t , ie 2520 is a factor of t . Since $t < 5000$, this means $t = 2520$. So in the bottom paddock, we have $2520/120 = 21$ merinos, $2520/56 = 45$ suffolks and $2520/45 = 56$ border leicesters. These three breeds account for $21 + 45 + 56 = 122$ of the sheep in the bottom paddock. The remaining one third are corriedales, so 122 is two thirds of the sheep in the bottom paddock. Thus the number of sheep in the bottom paddock is $122 \times 3/2 = 183$.
3. (a) Start by drawing up a table

y	$y/2$	$1/(1+1/y)$
1/2	1/4	$1/(1+1/2) = 2/3$
1/4	1/8	$1/(1+1/4) = 4/5$
2/3	1/3	$1/(1+2/3) = 3/5$
1/8	1/16	$1/(1+1/8) = 8/9$
4/5	2/5	$1/(1+4/5) = 5/9$

All the numbers in the table are in C , thus $5/9$ is in C .

(b) To show that $15/37$ is in C , try to work backwards: $15/37$ could have come from $2 \times 15/37 = 30/37$ or x where $1/(1+x) = 15/37$, ie $1+x = 37/15$, so $x = 37/15 - 1 = 22/15$. But $22/15$ is not between 0 and 1, so $22/15$ is not in C . So if $15/37$ is in C , it must have come from $30/37$. Can we show $30/37$ is in C ? We continue like this. It is easiest to make a table. For each number y in the first columns, y could have come from $2y$ or from x where $1/(1+x) = y$, ie $x = 1/y - 1$.

y	$2y$	$1/y - 1$	$0 < 2y < 1?$	$0 < 1/y - 1 < 1?$
14/15	28/15	$14/14 - 1 = 1/14$	N	Y
1/14	1/7	$14/1 - 1 = 13$	Y	N
1/7				
2/7				
4/7				
3/4
1/3				
2/3				

Since we are told that $1/2$ is in C , tracing back through the table shows that $15/37$ is also in C .