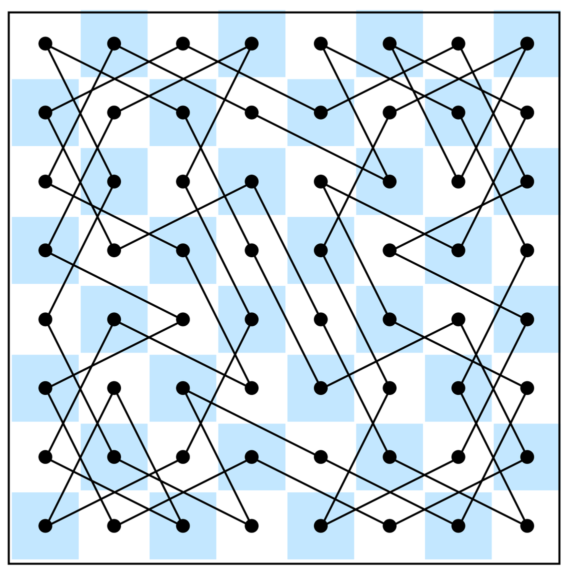
Hamilton Cycle

A hamilton cycle passes through every vertex of a graph exactly once.

e.g.

The Knight's Tour Problem:

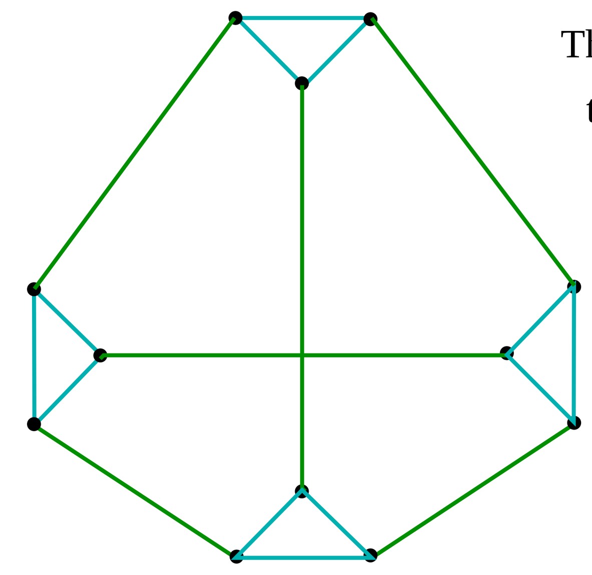
Can a knight visit every square of a chessboard in a round trip?



50	11	24	63	14	37	26	35
23	62	51	12	25	34	15	38
10	49	64	21	40	13	36	27
61	22	9	52	33	28	39	16
48	7	60	1	20	41	54	29
54	4	45	8	53	32	17	42
6	47	2	57	44	19	30	55
3	58	5	46	31	56	43	18

Here is a solution which also produces a magic square.

Each row and column add to 260.



The Cayley graph of the tetrahedral group generated by r and f where

$r^3 = 1 = f^2$

$(rf)^3 = 1$

r

f

Group

A group is a set of elements and a consistent* way of combining them. e.g. {0,1,2,3,4} with addition mod 5.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

* satifying closure, associativity, identity and unique inverses

Cayley Graph

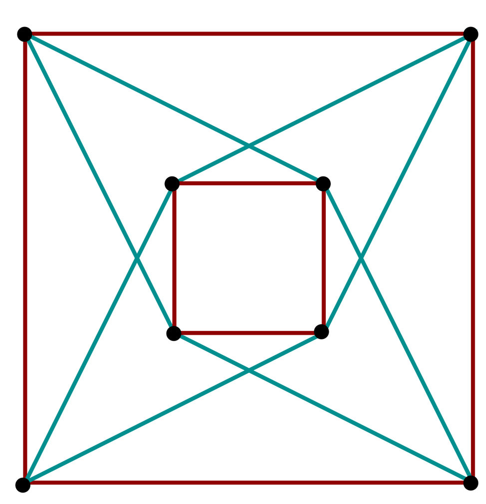
A Cayley graph has vertices and edges. It has a vertex for each element of a group. An edge shows two elements are related by some given element called a generator. Cayley graphs can be drawn with directed or undirected edges.

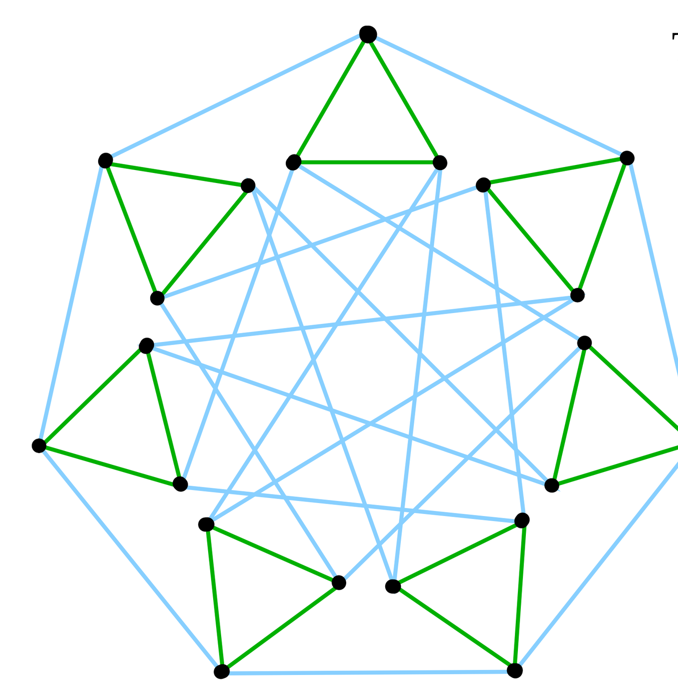
The Cayley graph of the Quartenion Group generated by a and b

$a^2 = b^2 = (ab)^2$

a

b





The Cayley graph of a group with 21 elements and generators s and t

$s^3 = 1$

$sts^{-1} = t^2$

s

t

Hamilton cycle decomposition of Cayley graphs

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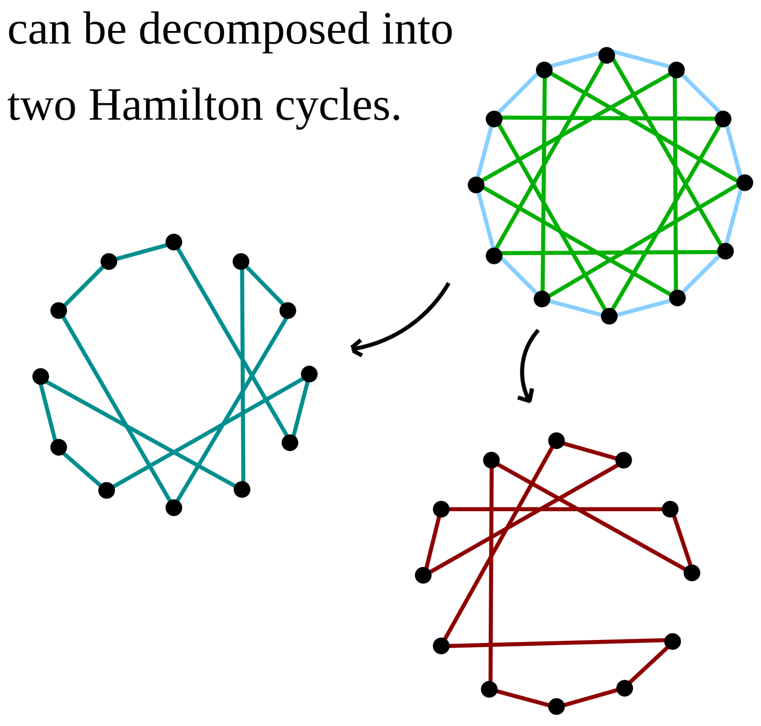
The Cayley graph on the addition group mod 12 with generators 1 and 4 can be decomposed into two Hamilton cycles.



Alspach's conjecture

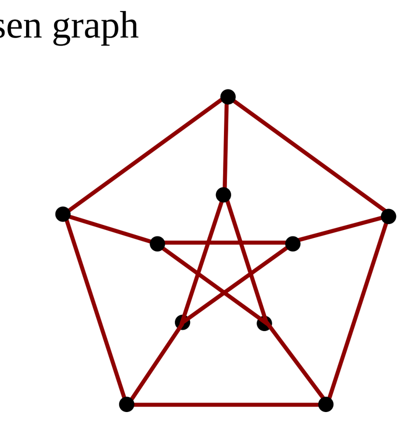
Every 2k−regular connected Cayley graph on a finite commutative group has a hamilton cycle decomposition.

Does every connected Cayley graph have a hamilton cycle?

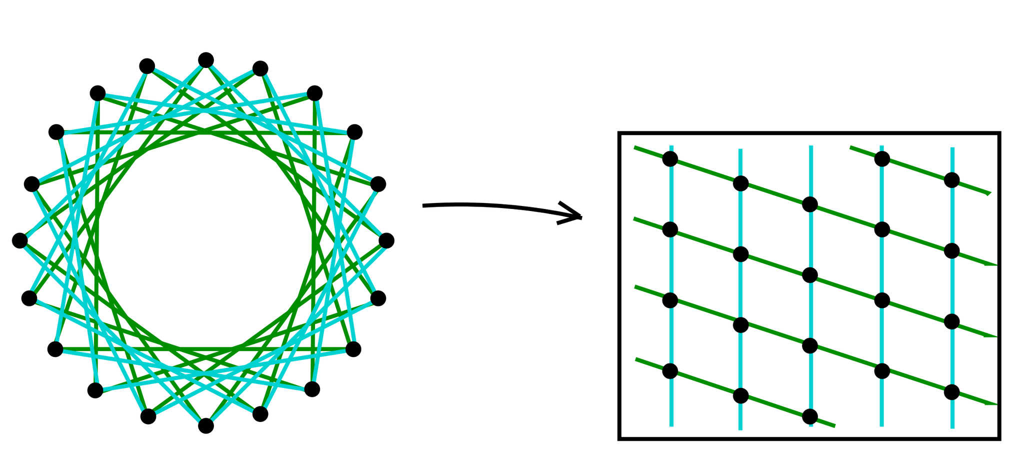
...for commutative groups, yes.

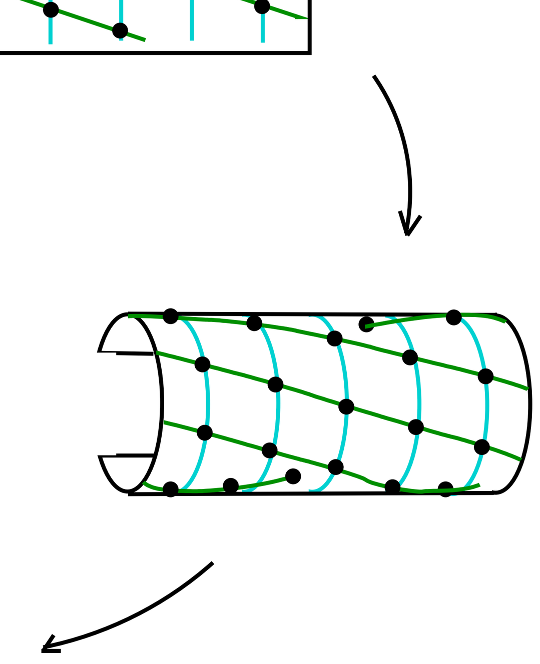
...for non−commutative groups this is not known.

In the wider class of vertex transitive graphs, the Petersen graph does not have a Hamilton cycle.

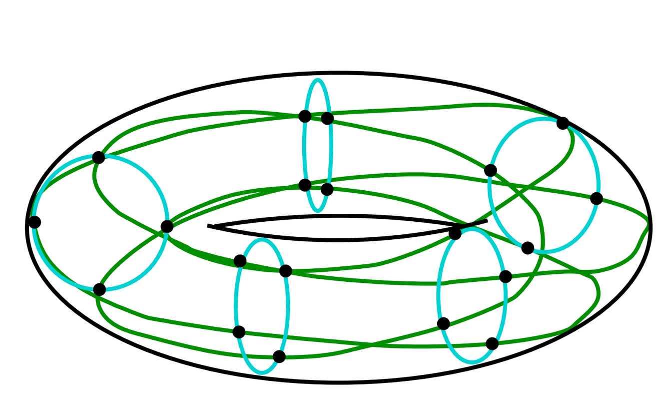


Alspach's conjecture was proven true for the case k = 2 by Bermond, Favarn and Maheo (1989).





Cayley graphs for addition mod 20 generated by 5 and 6.



Our Results

Theorem
All 6−regular Cayley graphs which have a generator of the same order as the group are decomposable into hamilton cycles.

Theorem
All 6−regular Cayley graphs on an odd cyclic group are decomposable into hamilton cycles.

References

[1] R. Wilson and J. Watkins, *Graphs, an Intoductory Approach,* John Wiley & Sons. Inc, New York 1990

[2] C Berge,*Graphs,* North−Holland, Amsterdam 1991

[3] I. Grossman and W Magus, *Groups and their Graphs,* Mathematical Association of America, 1964

[4] S. Curran and J. Gallian, *Hamilton cycles and paths in Cayley graphs and digraphs − A survey,* Discrete Mathematics 156 (1996) 1−18.