STAT4404: Advanced Stochastic Processes II, Semester 1, 2013. Quiz 2

Exercise 1:

Let $X_1, X_2, \ldots$ be a sequence of non-negative independent random variables and consider $N(t) = max\{n : X_1 + X_2 + \ldots + X_n \leq t\}$. Define an appropriate filtration and show that $N(t) + 1$ is a stopping time with respect to the filtration.

Exercise 2:

Let W be a standard Wiener process and define $S(t) = e^{at+bW(t)}$. Using the properties of the standard Wiener process and assuming that $E|S(t)| < \infty$. Show that S is a martingale with respect to the filtration generated by W if and only if $a + \frac{1}{2}b^2 = 0$ and in this case $E(S(t)) = 1$.

Exercise 3:

Let $\{S_n : n \geq 0\}$ be a simple symmetric random walk with $0 < S_0 < N$ with absorbing barriers at 0 and N. If conditions hold use the optional stopping theorem to show that the mean time until absorption is $E(S_0(N - S_0))$.
Hint: If needed you might assume that $\{S_n^2 - n : n \geq 0\}$ *is a martingale.*