STAT4404: Advanced Stochastic Processes II, Semester 1, 2013. Take Home Assignment 4, Due: April 22, 2013, 4:00PM.

Exercise 1:

If T_1 and T_2 are stopping times with respect to a filtration $\mathcal{F}$, show that $T_1 + T_2$, $max\{T_1, T_2\}$ and $min\{T_1, T_2\}$ are also stopping times.

Exercise 2:

Let D be a diffusion with instantaneous mean and variance $a(t, x)$ and $b(t, x)$ and let $M(t, \theta) = E(e^{\theta D(t)})$, the moment generating function of $D(t)$. Use the forward diffusion equation to derive Bartlett's equation:

$$\frac{\partial M}{\partial t} = \theta a(t, \frac{\partial}{\partial \theta})M + \frac{1}{2}\theta^2 b(t, \frac{\partial}{\partial \theta})M$$

where

$$g(t, \frac{\partial}{\partial \theta})M = \sum_n \gamma_n(t) \frac{\partial^n M}{\partial \theta^n}$$

if $g(t, x) = \sum_{n=0}^{\infty} \gamma_n(t) x^n$.

Exercise 3:

Write down Bartlett's equation in the case of the Wiener process D having drift m and instantaneous variance 1, and solve it subject to the boundary condition $D(0) = 0$.
Hint: Use previous exercise.

Exercise 4:

Let W be a standard Wiener process and let $X(t) = exp\{i\theta W(t) + \frac{1}{2}\theta^2 t\}$ where $i = \sqrt{-1}$. Show that X is a martingale with respect to the filtration given by $\mathcal{F}_t = \sigma(\{W(u) : u \leq t\})$