MATH4401: Advanced Analysis

First Semester 2005
9/5/2005

Assignment Number 5

Problem 1 Suppose Ω is an open, connected subset of $\mathbb{R}^n$, and $p \in [1, \infty]$ is fixed. Suppose $u \in W^{1,p}(\Omega)$ satisfies $Du = 0$ a.e.. Show that u is constant a.e..

Problem 2 Consider $f : \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (x^2 - y^2 - x, 2xy - y)$.
Calculate $\mathrm{d}(f, B_3((0,0)), (0,0))$.

Problem 3 **a)** Let X be a topological space, $x_0 \in X$, and let X_0 be the path-component of X which contains x_0. Show that the groups $\pi_1(X, x_0)$ and $\pi_1(X_0, x_0)$ are isomorphic. (Hint: The inclusion $j : X_0 \to X$ induces a map $j_* : \pi_1(X_0, x_0) \to \pi_1(X, x_0)$ via $[f] \mapsto [jf]$. Show that j_* is an isomorphism.)

b) Let X be a path-connected topological space, with $x_0, x_1 \in X$. Show that the groups $\pi_1(X, x_0)$ und $\pi_1(X, x_1)$ are isomorphic. (This means that one can speak of $\pi_1(X)$ in this situation.)

Due: Tuesday, 25/5/2005 before the lecture

Current assignments will be available at
`http://www.maths.uq.edu.au/courses/MATH4401/Tutorials.html`