MATH4401: Advanced Analysis

Assignment Number 4

Problem 1 Prove *Minkowski's inequality*: for $1 \le p \le \infty$ and $u, v \in L^p(\Omega)$ (Ω a domain in $\mathbb{R}^n$, $n \ge 1$): there holds:

$$||u+v||_{L^p(\Omega)} \le ||u||_{L^p(\Omega)} + ||v||_{L^p(\Omega)} .$$

Problem 2 Let B be the open unit ball in $\mathbb{R}^n$, $n > 1$. Show that the following function is unbounded, but lies in $W^{1,n}(B)$:

$$x \mapsto \log(\log(1 + \tfrac{1}{|x|})).$$

Problem 3 Let X be a topological space, and set $I := [0,1]$. The *inverse* of a path f in X (notation as in Assignment 2) is defined by

$$f^{-1}(t) = f(1-t) \quad t \in I .$$

The *constant path* in $p \in X$ is defined via $i_p(t) = p$ for all $t \in I$.
a) Does there always hold $f * f^{-1} = f^{-1} * f$?
b) Let f be a path in X with $f(0) = f(1) = p$, (a *loop with basepoint* p). Does there hold $f * f^{-1} = f^{-1} * f = i_p$?
c) The *path class* of a path$f$, $[f]$, is the equivalence class of f w.r.t. $\simeq_{\partial I}$. In particular, $[f](0)$, $[f](1)$ are well defined.
Fix $p \in X$. Show that the set

$$\{[f] \,:\, [f] \text{ is a path class with } [f](0) = [f](1) = p\}$$

is a group with the binary operation $[f][g] := [f * g]$, identity $[i_p]$ and $[f]^{-1} := [f^{-1}]$. The group is called the *fundamental group of x with basepoint p*, and is denoted $\pi_1(X, p)$.

Due: Thursday, 5/5/2005 before the tutorial

Current assignments will be available at
`http://www.maths.uq.edu.au/courses/MATH4401/Tutorials.html`