MATH4401: Advanced Analysis First Semester 200[illegible]
2/3/2004

Assignment Number 1

Problem 1 Let X be a topological space, $A \subseteq X$. Show that there holds:
a) A is open iff $\operatorname{Int} A = A$;
b) A is closed iff $\overline{A} = A$.

Problem 2 Let $X$and Y be topological spaces, $I = [0,1]$. Functions $f,\ g \in C(X,Y)$ are called *homotopic* (i.e., $f \simeq g$) if there exists a *homotopy* between f and g, i.e., $F \in C(I \times X, Y)$ with $F(0,\cdot\,) = f(\,\cdot\,)$, $F(1,\cdot\,) = g(\,\cdot\,)$.
a) Show that $\simeq$ is an equivalence relationship on $C(X,Y)$, the space of continuous maps from X to Y.
b) Let $A \subseteq X$. Maps f and g in $C(X,Y)$ with $f\big|_A = g\big|_A$ are called *homotopic relative to* A (written: $f \simeq_A g$, or $f \simeq g \operatorname{rel} A$), if there is a homotopy F between f and g that further satisfies: $F(\,\cdot\,, a) = f(a)\ \ (= g(a))$ for every $a \in A$. Show that $\simeq_A$ is also an equivalence relation on $C(X,Y)$.
c) Now consider $X = I$, $A = \partial I$. A *path* in Y is a map in $C(I,Y)$. The *composition* of two paths from f and g in Y is defined by:

$$(f * g)(t) = \begin{cases} f(2t) & \text{for } 0 \le t < \frac{1}{2} \\ g(2t-1) & \text{for } \frac{1}{2} \le t \le 1. \end{cases}$$

The composition $f * g$ is obviously a path iff $f(1) = g(0)$. Now consider paths f_1, f_2, g_1 und g_2 in Y with $f_1 \simeq_{\partial I} f_2$, $g_1 \simeq_{\partial I} g_2$. Show that $f_1(1) = g_1(0)$ implies: $f_1 * g_1 \simeq_{\partial I} f_2 * g_2$.

Problem 3 Let (X,d) be a metric space, $A \subseteq X$. Let $d(x,A)$ denote the distance from a point $x \in X$ to A, i.e. $d(x,A) = \inf\limits_{a \in A} d(x,a)$. Show:
x is an accumulation point of $A \Leftrightarrow d(x, A\backslash\{x\}) = 0 \Leftrightarrow$ there exists a sequence $\{x_i\}$ in A with limit x and $x_i \neq x$ for all $i \in \mathbb{N}$.

Due: Thursday, 17/3/2004 before the tutorial

Current assignments will be available at
`http://www.maths.uq.edu.au/courses/MATH4401/Tutorials.html`