MATH 3500 Module 1 2013

Problems for tutorials

① Classify as open, closed or neither:

a) $\{x \in \mathbb{R}^n : x_1 \geq x_2\}$ b) $\{x \in \mathbb{R}^n : \frac{1}{2} < |x| \leq 1\}$

② Calculate all second partials of the following functions on $\mathbb{R}^2$, & verify $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ in each case:

a) $\sin(xy)$ b) e^{x+y} c) $\frac{1}{1+x^2}$

③ Write the following in multiindex notation:

a) $\frac{\partial^3 f}{\partial y^3}$ (coords x, y, z in $\mathbb{R}^3$)

b) $\frac{\partial^3 f}{\partial y^3}$ (coords x, y in $\mathbb{R}^2$)

c) $\frac{\partial^{17} f}{\partial x_1 \partial x_2 \partial x_7^{14} \partial x_{10}}$ (coords $x_1, \ldots, x_{12}$ in $\mathbb{R}^{12}$)

④ Classify the following PDE's (linear/semilinear/quasilinear/fully nonlinear, & give their order):

a) $u_t + u_{xxx} = 0$ (Airy's eq^n.)

b) $u_t + uu_x = 0$ (Inviscid Burger's eq^n.)

c) $-\Delta u = \lambda u$ Helmholtz's equation.

*⑤ Let $u : \mathbb{R}^n \setminus \{0\} \to \mathbb{R}^n$, $n \geq 2$, be given by $u(x) = x/|x|$.
Show that u satisfies $\Delta u + u|Du|^2 = 0$ ⊕
Here $|Du|^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} (D_i u^j)^2$, and note that ⊕ is a system: $\Delta u^i + u^i \cdot |Du|^2 = 0$, $i = 1, \ldots, n$.