MATH3500 Asst. I Sol^{n}s

(P1) a) (i) Highest order derivative is u_{xxx}, so 3rd order. Coeff of u_{xxx} is 1; coeff of u_{xx} is u^2, which is not a f^{n} only of x. Hence 3rd order, semi-linear.

(ii) Highest order derivative is Δu, so 2nd order: ~~this~~ derivative occurs in a nonlinear fashion ($(\Delta u)^2$), so 2nd order fully nonlinear.

(iii) Highest order derivative is u_{xxx}, so 3rd order. Co-eff of u_{xxx} is $(u_t + u)$, which is a function of x & derivatives of order $\leq 2 = 3-1$, so 3rd order quasi-linear.

(P2) ($\Rightarrow$) $u_t = u_{xx}$ for $u(x,t) = v(p)$

Note $p > 0 \iff t > 0$ and $x \neq 0$.

Note further $p_x = {}^{2x}/_{t}$, $p_t = {}^{-x^2}/_{t^2}$

So $u_t = \dfrac{\partial v}{\partial t} = v' p_t = \dfrac{-x^2}{t^2} v'$,

$u_x = \dfrac{\partial v}{\partial x} = v' p_x = \dfrac{2x}{t} v'$ (*)

$\Rightarrow u_{xx} = (u_x)_x = \left(\dfrac{2x}{t} v'\right)_x$

$= \dfrac{2x}{t}(v')_x + \dfrac{2}{t} v'$ (**)

By (*) with v' in place of v we see $(v')_x = \dfrac{2x}{t} v''$.

Hence (**) $\Rightarrow u_{xx} = \dfrac{4x^2}{t^2} v'' + \dfrac{2}{t} v'$.

So $u_t = u_{xx} \Rightarrow \quad -\frac{x^2}{t^2} v' = \frac{4x^2}{t^2} v'' + \frac{2}{t} v'$

$\xRightarrow{\times t} \quad -\frac{x^2}{t} v' = \frac{4x^2}{t} v'' + 2v'$

i.e. $\quad -pv' = 4pv'' + 2v'$ i.e.

$4\,'' + (2+p)v' = 0 \quad \oplus$ as req^d.

($\Leftarrow$) If $\oplus$ holds for all $p > 0$, we can completely reverse the above argument (since $p > 0 \Rightarrow$ in particular $t > 0$) to see $u_t = u_{xx}$. $\square$