MATH3500: Problems & Applications in Modern Mathematics

Second Semester 2013
23/08/2013

Assignment Number 1

Problem 1 (a) Classify the following partial differential equations according to the scheme given in class (i.e., give their order; and further classify them as linear, semi-linear, quasi-linear or fully non-linear). Explain your answers. **(10 marks)**

(i) $u_t + u_{xxx} + u^2 u_{xx} = 0$ for $u(x,t)$ defined on $\mathbb{R} \times (0,\infty)$,

(ii) $(\Delta u)^2 = f$ for $u(x)$ defined on $\mathbb{R}^n$, $n \geq 2$, and $f(x)$ defined on $\mathbb{R}^n$,

(iii) $(u_t + u)(u_{xxx} + u_{xx}) = 0$ for $u(x,t)$ defined on $\mathbb{R} \times (0,\infty)$.

(b) Consider the one-dimensional heat equation

$$u_t = u_{xx} \tag{1}$$

on $\mathbb{R} \times (0,\infty)$, for the unknown function $u(x,t)$.

Show that $u(x,t) = v(x^2/t)$ satisfies (1) if and only if $v(p)$ satisfies

$$4pv''(p) + (2+p)v'(p) = 0 \tag{2}$$

for $p > 0$. **(40 marks)**

Problem 2 *(50 points)* Write a short essay of 300-600 words about one of the equations or systems listed below. Your report should introduce the equation, classify it, carefully explain the setting (e.g. on what sort of domain do we try to solve the equation? what sorts of boundary value problems do we consider?) and provide an explanation of where the equation can be applied (if relevant). You may also wish to provide a discussion of numerical approaches (if relevant), or discussions of existence, uniqueness and well posedness. You may also wish to mention extensions and/or generalizations. (You can, of course, give a bit of historical background, but don't just write about the history).

Your report needs to include at least one relevant diagram or illustration. You must provide references. Your references must include at least one *published* journal article, and one book. Citations need to be correct. The word limit does not include diagrams.

The candidates: Monge-Ampère equation; Minimal surface equation; Fokker-Planck equation; Biharmonic equation; Burgers' equation; Porous medium equation; Eikonal equation; Klein-Gordon equation; sine-Gordon equation; the equations of linear elasticity; Ginzburg-Landau equation.

Due: 12 noon, Friday, 23/08/2013. This assignment represents 10% of your marks for the course.

Current assignments will be available at
`http://www.maths.uq.edu.au/courses/MATH3401/Tutorials.html`