Aim: well/ill-posedness.

Topology basics

$x = (x_1, \cdots, x_n) \in \mathbb{R}^n \quad n \geq 1.$

$|x| \left(= \|x\|\right) = \left(\sum_{i=1}^n x_i^2\right)^{1/2}$

$B_r(x)$ = ball of radius r centred at x

$= \{y \in \mathbb{R}^n : |x-y| < r\}.$

$\Omega \subset \mathbb{R}^n$

$\mathrm{Int}\,\Omega$ = in

$= \{x \in \mathbb{R}^n$

$: B_\varepsilon(x) \subset$

$\mathrm{Ext}\,\Omega = \{$

$: B_\varepsilon(x) \cap \Omega$

$\partial\Omega$ = boun

$= \{x \in \mathbb{R}^n :$

$B_\varepsilon(x) \cap \Omega \neq$

S.

$\Omega \subset \mathbb{R}^n \quad (\subseteq)$

$\mathrm{Int}\,\Omega$ = interior of Ω

$= \{x \in \mathbb{R}^n : \exists\, \varepsilon > 0 : B_\varepsilon(x) \subset \Omega\}.$

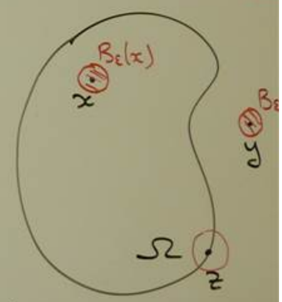
$\mathrm{Ext}\,\Omega = \{x \in \mathbb{R}^n : \exists\, \varepsilon > 0 : B_\varepsilon(x) \cap \Omega = \emptyset\}.$

$\partial\Omega$ = boundary of Ω

$= \{x \in \mathbb{R}^n :$ for <u>any</u> $\varepsilon > 0$,

$B_\varepsilon(x) \cap \Omega \neq \emptyset$ &

$B_\varepsilon(x) \cap \underbrace{(\mathbb{R}^n \setminus}_{\Omega^c}$

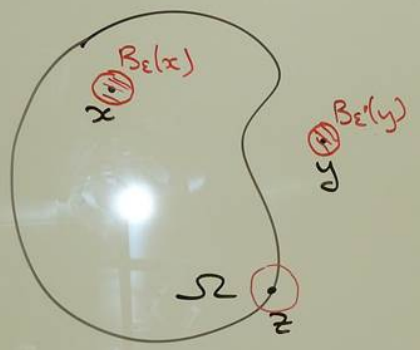


$\mathbb{R}^n \setminus \Omega$

Ω is open ⇔

Ω is closed

$$B_\varepsilon(x) \cap \underbrace{(\mathbb{R}^n \setminus \Omega)}_{\Omega^c \,=\, \text{complement of } \Omega} \neq \emptyset \}.$$



$x \in \operatorname{Int} \Omega$

$y \in \operatorname{Ext} \Omega$

$z \in \partial\Omega$.

$\mathbb{R}^n \setminus \Omega$

Ω is open $\Longleftrightarrow \operatorname{Int}\Omega = \Omega$.

Ω is closed $\Longleftrightarrow \Omega^c$ is open.

Easy to show:

⊛ Ω is open $\Longleftrightarrow \partial\Omega \cap \Omega = \emptyset$.

⊛ Ω is closed $\Longleftrightarrow \partial\Omega \subset \Omega$.

Rmk:

① a set may neither open nor closed.

$(1,2]$.

② clos
is
only
Ω
con
m

(2) $\mathbb{R}^n$ is both open & closed (clopen), as is $\emptyset$. These are the only clopen sets in $\mathbb{R}^n$.

Ω is called <u>path(wise) connected</u> if any two points in Ω can be connected by

a pat

a path in Ω.

connected.

connected.

disconnected

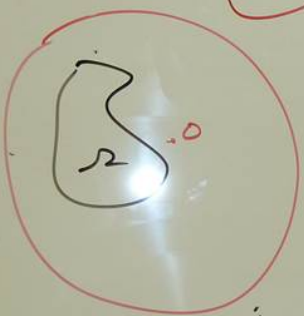
Ω is connected $\Leftrightarrow$ if $\Omega \subset \Omega_1 \cup \Omega_2$ with Ω_1, Ω_2 open & disjoint, then $\Omega_1 = \emptyset$ or $\Omega_2 = \emptyset$

In general, path connected $\Rightarrow$ connected.

Path connected $\Leftrightarrow$ connected for open sets in $\mathbb{R}^n$.

Ω is
$\Omega \subset B_r($
Finite
NO. E.g
Has ze

Ω is bounded if $\exists r>0$: $\Omega \subset B_r(0)$. $=(0,\dots,0)\in\mathbb{R}^n$.

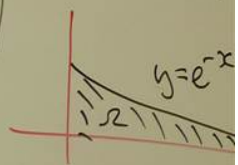


Note: bded $\Rightarrow$ finite volume ($\mathrm{vol}(\Omega) \leq \mathrm{vol}(B_r(0)) < \infty$).

Finite volume $\Rightarrow$? bded.

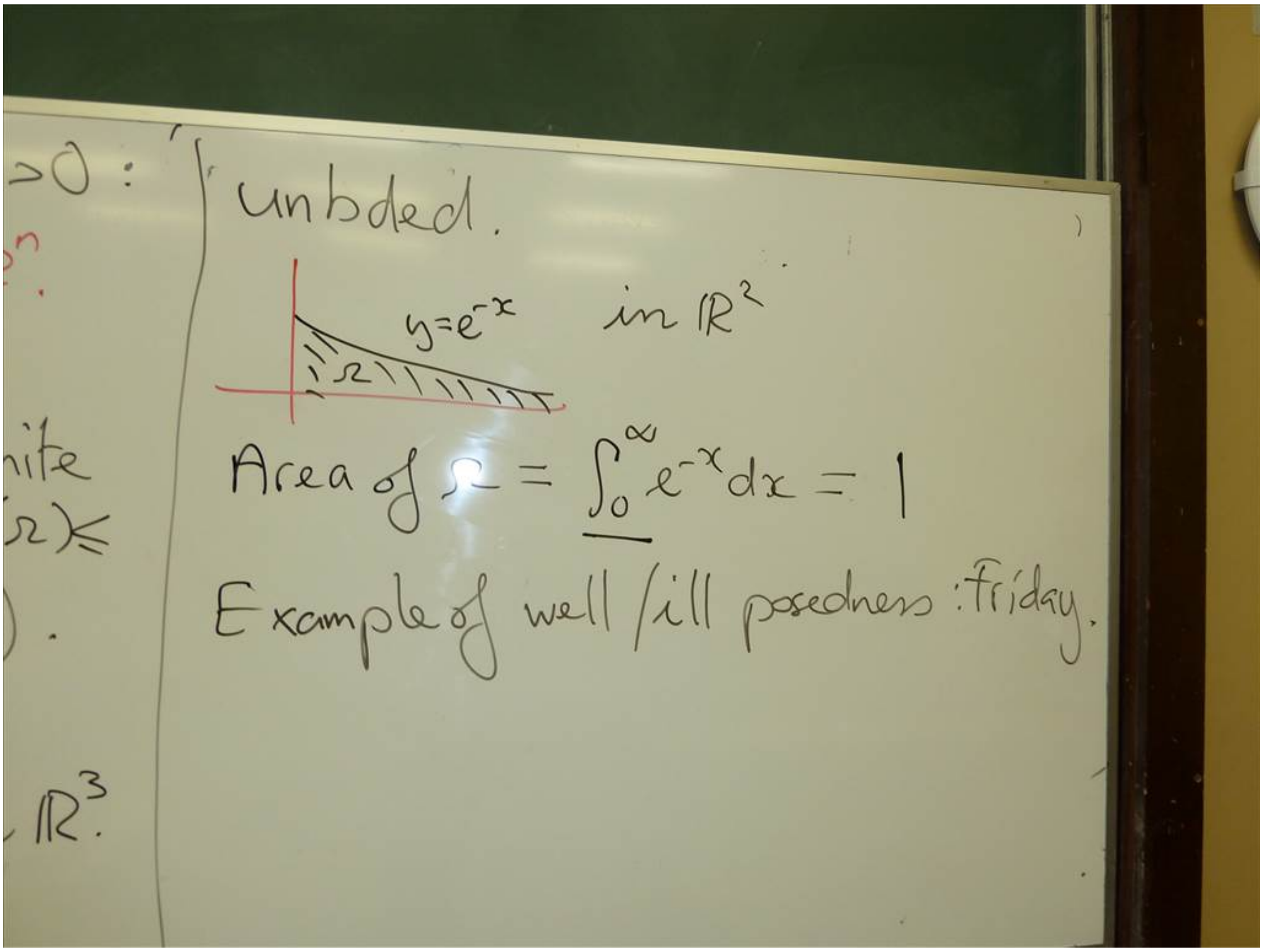
NO. E.g. $\Omega =$ x axis in $\mathbb{R}^3$.
Has zero volume, but is

unbded.



Area of Ω

Example of


unbded.
$y=e^{-x}$ in $\mathbb{R}^2$
Ω
Area of $\Omega = \int_0^\infty e^{-x}\,dx = 1$
Example of well/ill posedness: Friday.