Examples of well posedness: later

Examples of ill posedness:

1. Any domain Ω:

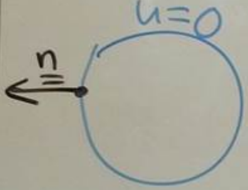
$$\Delta u = 0 \quad \text{in } \Omega.$$

for any $C_1 \in \mathbb{R}$, $u \equiv C_1$ (i.e., $u(x) = C_1 \ \forall x \in \Omega$) is a sol^n.

(indeed, $u = \sum_{j=1}^{n} a_j x_j + C_1$, $a_j \in \mathbb{R}$, is a sol^n).

* non-uniqueness.

② $\Omega = B_1(0)$ in $\mathbb{R}^n$.



(P) $\begin{cases} \Delta u = 0 \\ u = 0 \text{ on } \partial\Omega \\ \dfrac{\partial u}{\partial \underline{\underline{n}}} = 1 \text{ on } \partial\Omega. \end{cases}$

$\underline{\underline{n}}$ = external unit normal (vector field).

$$\frac{\partial u}{\partial \underline{\underline{n}}} = \nabla u \cdot \underline{\underline{n}}.$$

No sol^n?

* non-existence.

(P) is called over-determined.

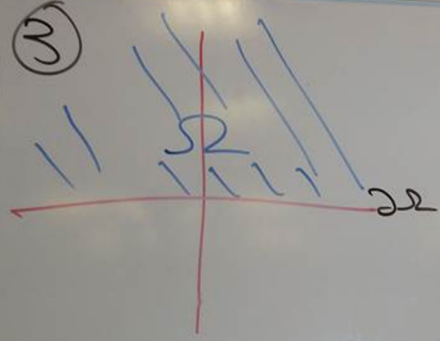
③

$\Omega = u$

Half P

in $\mathbb{R}$

③



$$(P)' \begin{cases} \Delta u = 0 \text{ in } \Omega \\ u(x,0) = \varepsilon \sin x \\ \frac{\partial u}{\partial y}(x,0) = 0. \end{cases}$$

Ω = Upper Half Plane in $\mathbb{R}^2$.

$\varepsilon \in \mathbb{R}$ fixed.

Sol^{n}: $u(x,y) = \varepsilon \sin x \cosh y$

Note on $\partial\Omega$

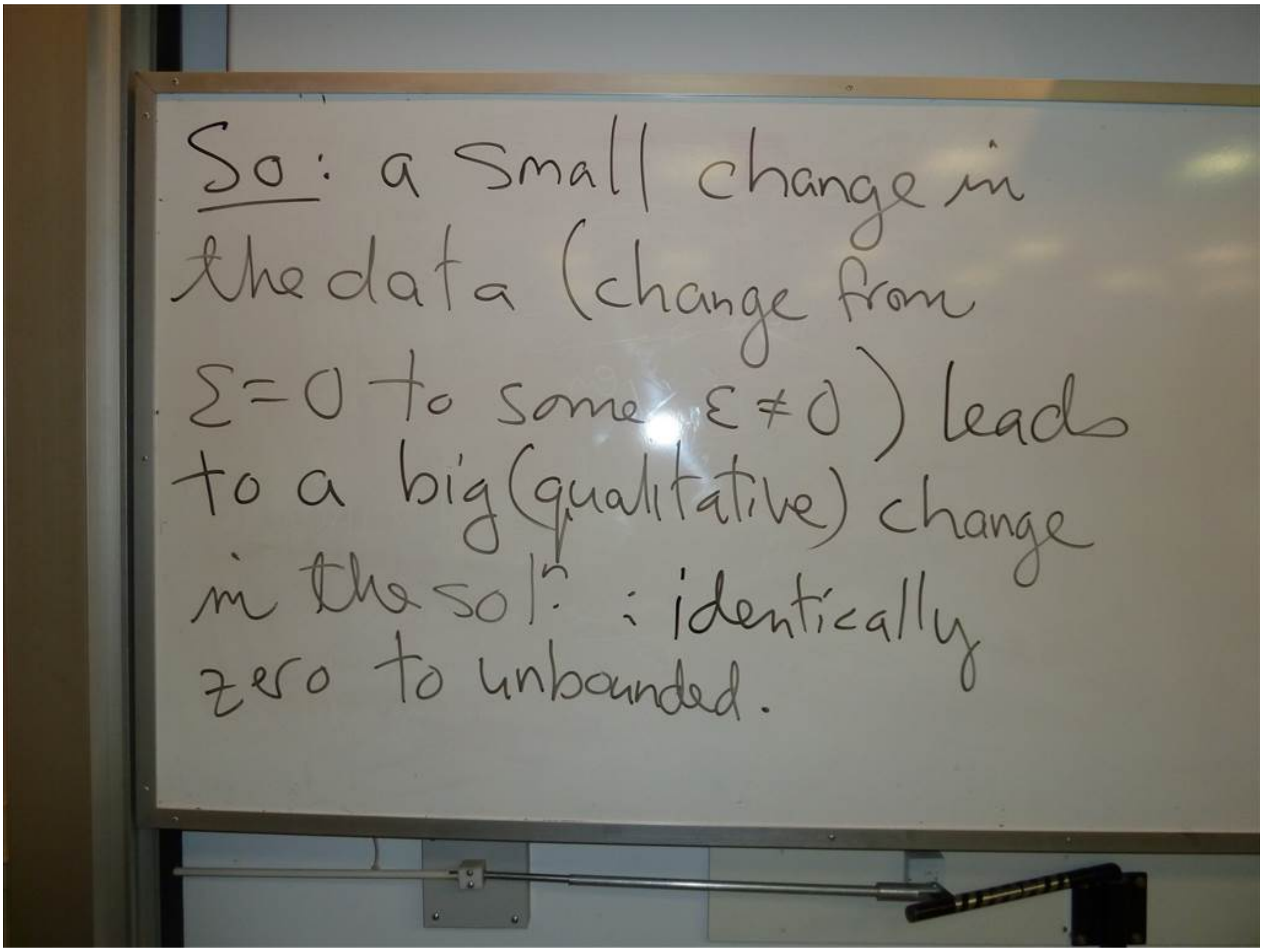
$|u| \leq \varepsilon$ &

$u_y = 0$.

But: $\lim_{y \to \infty} |u(x,y)| = \infty$

$\forall\, x \neq n\pi,\ n \in \mathbb{Z}$ for

any $\varepsilon \neq 0$.


So: a small change in
the data (change from
ε = 0 to some ε ≠ 0) leads
to a big (qualitative) change
in the soln: identically
zero to unbounded.