Partial Differential Eq^n's (PDE): a PDE is an eq^n involving an unknown f^n & 1 or more of its partial derivatives. (System: >1 equation, >1 unknown f^n).

E.g.① $\Delta u =$
Laplace's eq
(Sometimes ∇
(in 2-D : u
Here $u = u(x,$
the unknown f
on some region

^{n}'s
an
n f^n
ial
> 1
^{n}).

E.g.① $\Delta u = 0$

Laplace's eq^n

(Sometimes $\nabla^2 u = 0$)

(in 2-D : $u_{xx} + u_{yy} = 0$

Here $u = u(x,y)$ is the unknown f^n, defined on some region in $\mathbb{R}^2$:

in general,

$u_{x_1x_1} + \cdots + u_{x_nx_n}$

or

$u_{11} + \cdots + u_{nn} =$

Here $(x_1, \cdots, x_n)$

variables.

E.g.② $u_{tt} = \Delta$

Defined on $\mathbb{R}$

$u = u(x,t)$

In general, in n dimensions,

$$u_{x_1x_1}+\cdots+u_{x_nx_n}=0$$

or

$$u_{11}+\cdots+u_{nn}=0$$

Here $(x_1,\cdots,x_n)$ are the domain variables.

E.g. (2) $u_{tt}=\Delta u$. wave eq^n.

Defined on $\mathbb{R}^n\times\mathbb{R}$, i.e.

$u=u(x,t)$

$=\underline{\underline{x}}=(x_1,\cdots,x_n)$

Indeed, often defined only on $\mathbb{R}^n\times(0,\infty)$.

E.g. (3): $u_t=\Delta u$ heat eq^n

E.g. (4) $u_t+cuu_x+u_{xxx}=0$

$u=u(x,t)$, $x\in\mathbb{R}$, $t\in\mathbb{R}^+=(0,\infty)$

Korteweg de Vries eq^n

models shallow water waves

E.g. (5)
$$\begin{cases} u_x=v_y \\ u_y=-v_x \end{cases}$$

$u(x,y)$, $v(x,y)$ defined on $\mathbb{R}\times\mathbb{R}$.

Cauchy-Riemann eq^{n}s:

"$\Longrightarrow$" $f=u+iv$ is analytic.

fined

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eat

$+ u_{xxx} = 0$

eq^n

models shallow water waves

E.g (5) $\begin{cases} u_x = v_y \\ u_y = -v_x \end{cases}$

$u(x,y)$, $v(x,y)$ defined on $\mathbb{R}\times\mathbb{R}$

Cauchy-Riemann eq^{n}s:

"$\Rightarrow$" $f = u + iv$ is analytic.

The order of a PDE
highest order of any
occuring in the eq^n
Above e.g (1)-(3) 2nd
(4) 3rd order, (5) 1st or

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The order of a PDE is the highest order of any derivative occuring in the eq^n.

Above e.g (1)-(3) 2nd order,
(4) 3rd order, (5) 1st order system.

So, given a PDE

$$F(x, u, \text{some partials of } u) = 0 \quad \odot$$

What questions do we want to consider for $\odot$?

* Existence/uniqueness of sol^{n}s to $\odot$, subject to suitable initial &or boundary conditions;

* numerical so
of sol^{n}s;
* sanity tes
mathematical
physical expe
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initial/bdy c
$\Omega \subset \mathbb{R}^n$, $n \geq$
$x \in \Omega$, $u : \Omega \Rightarrow$
ary

* numerical sol^{n}/approximation of sol^{n}s;
* sanity test: compare mathematical sol^{n} to physical expectations.

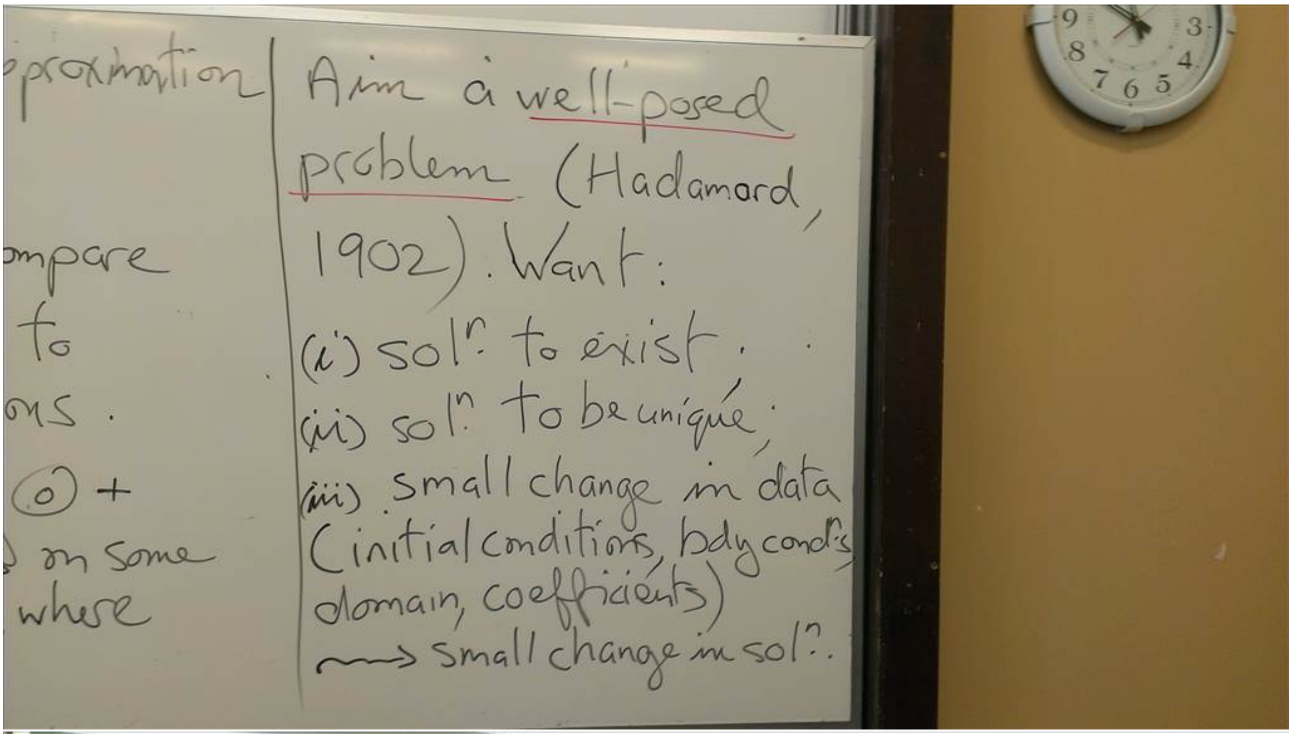
Consider solving $\odot$ + initial/bdy cond^{n}s on some $\Omega \subset \mathbb{R}^n$, $n \geq 2$, where $x \in \Omega$, $u : \Omega \Rightarrow \mathbb{R}$.

Aim a well-posed problem (Hadamard, 1902). Want:

(i) sol^{n} to exist;

(ii) sol^{n} to be unique;

(iii) small change in data (initial conditions, bdy cond^{n}s, domain, coefficients) $\rightsquigarrow$ small change in sol^{n}.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Aim a well-posed problem (Hadamard, 1902). Want:
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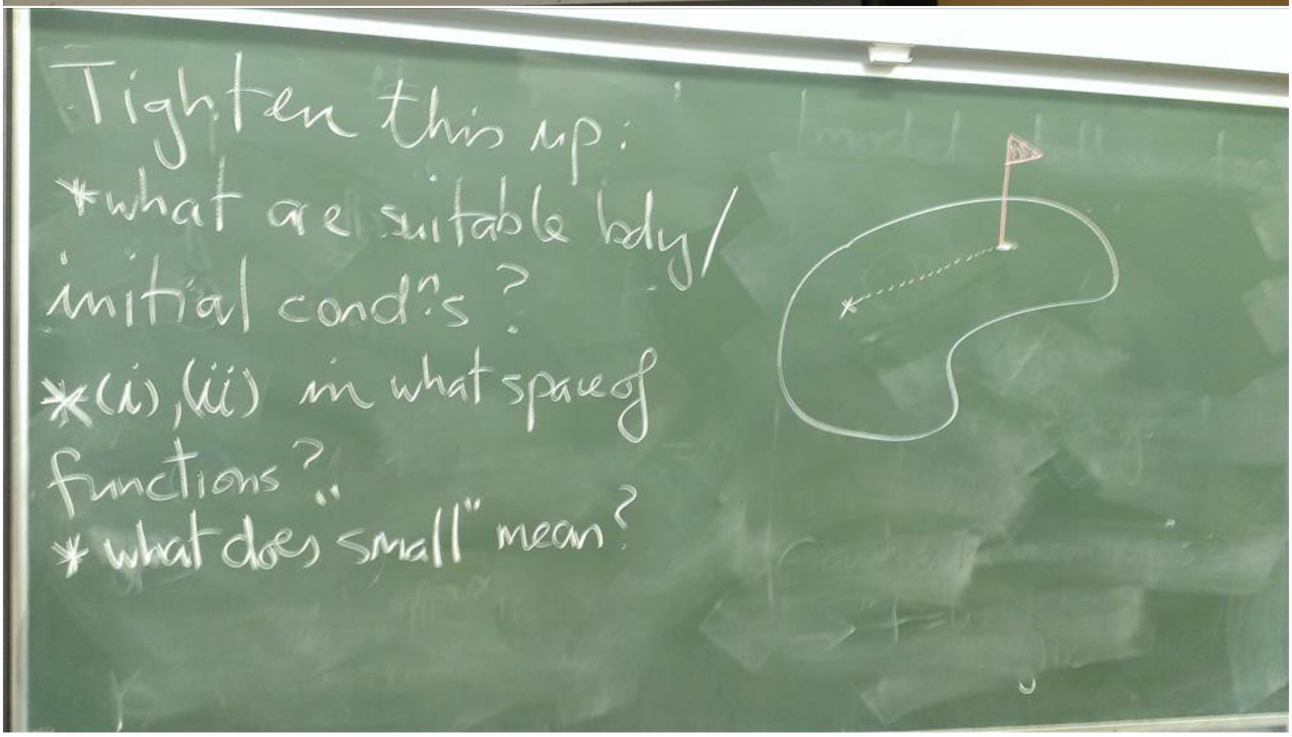
(iii) small change in data (initial conditions, bdy condⁿs, domain, coefficients) ⟶ small change in solⁿ.


Tighten this up:
* what are suitable bdy/initial condⁿs?
* (i), (ii) in what space of functions?
* what does "small" mean?