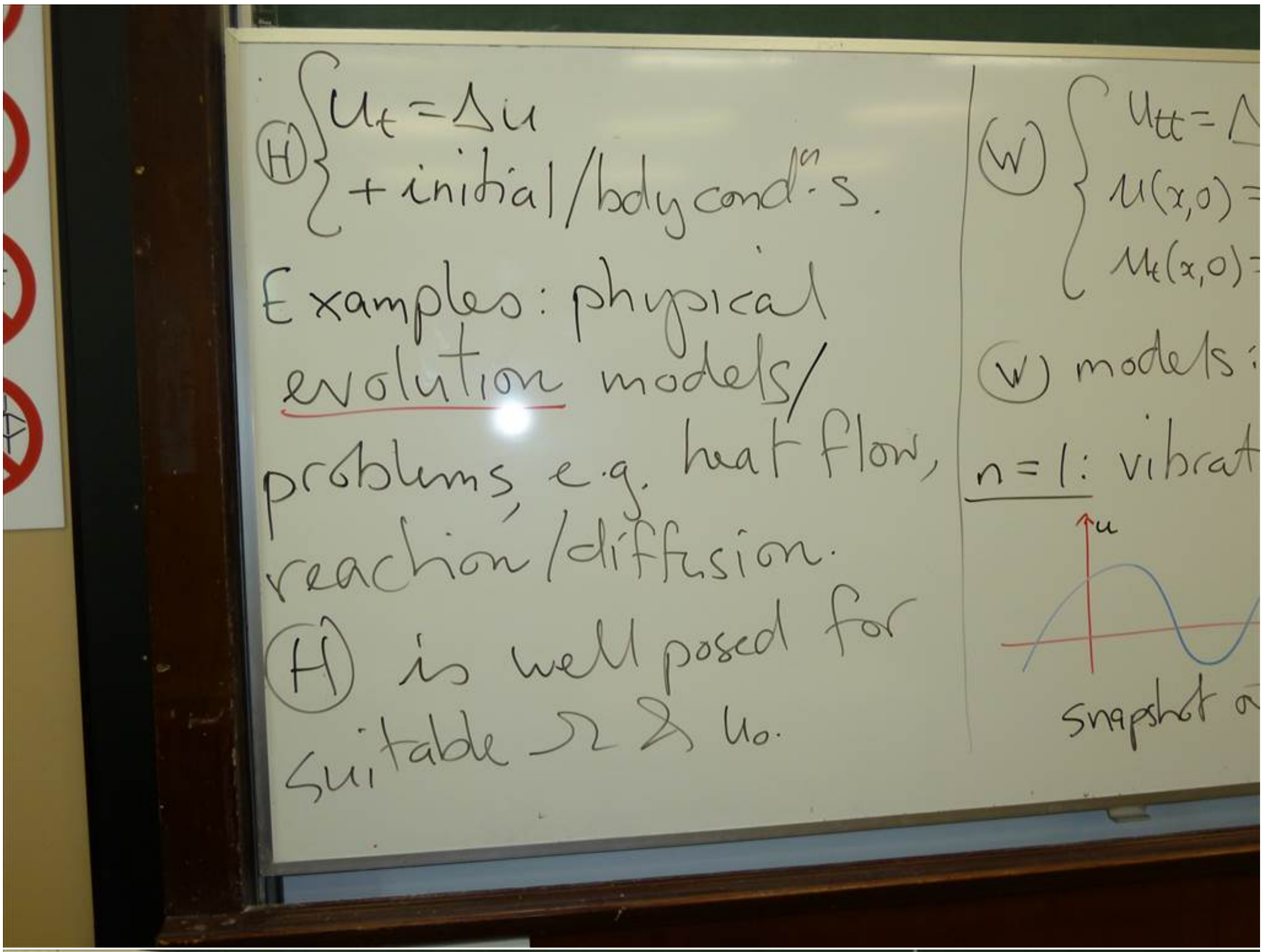
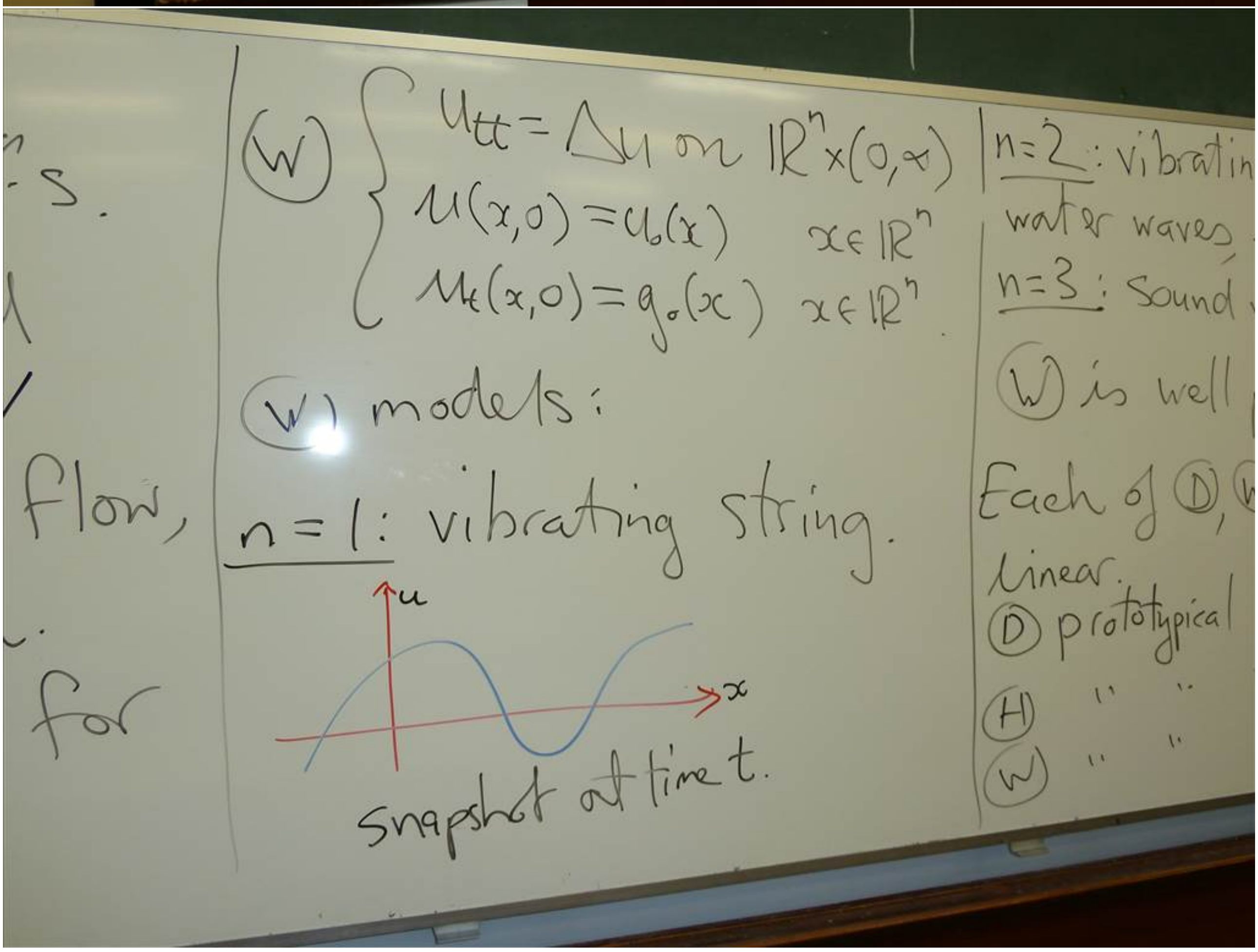
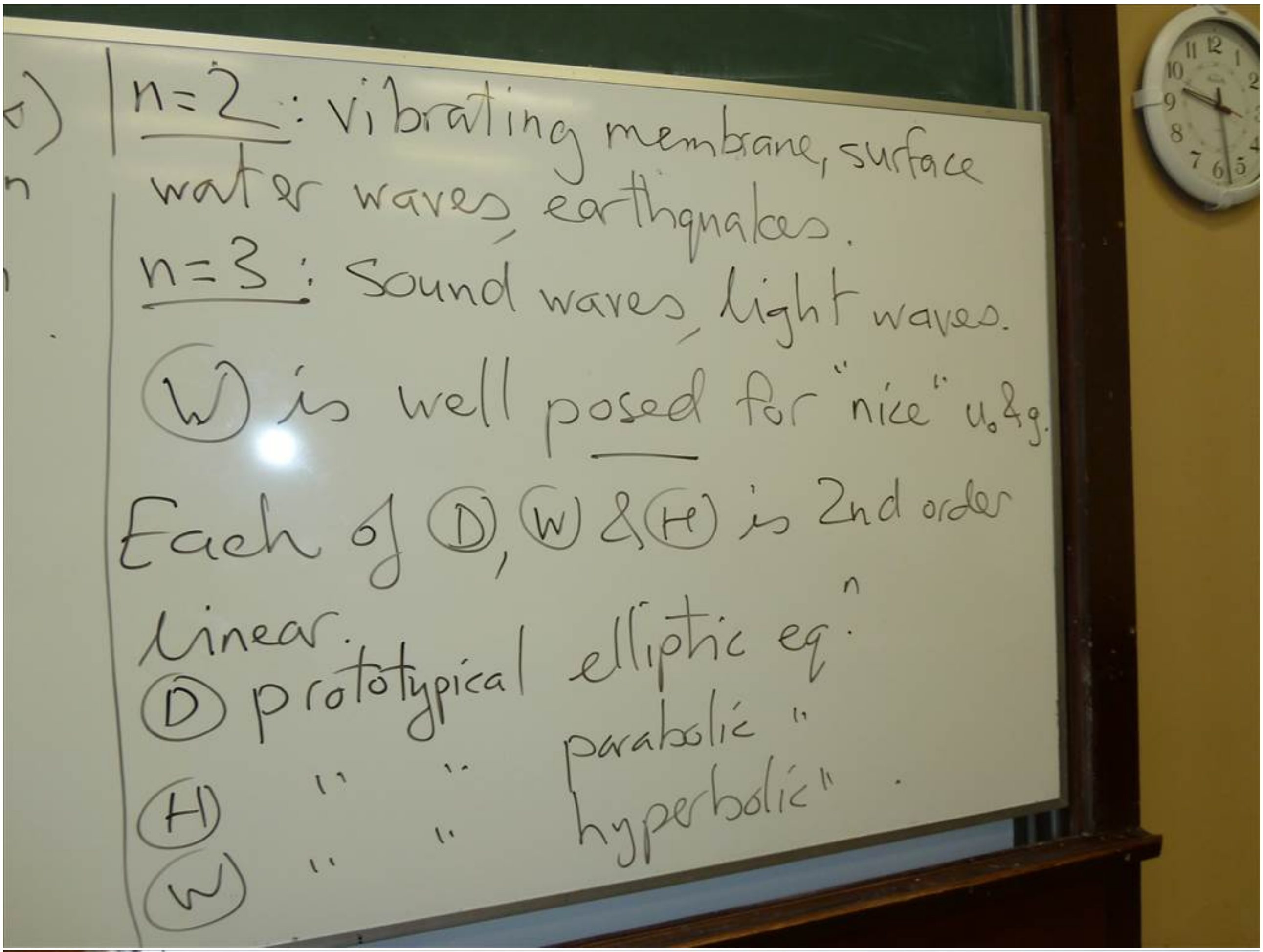
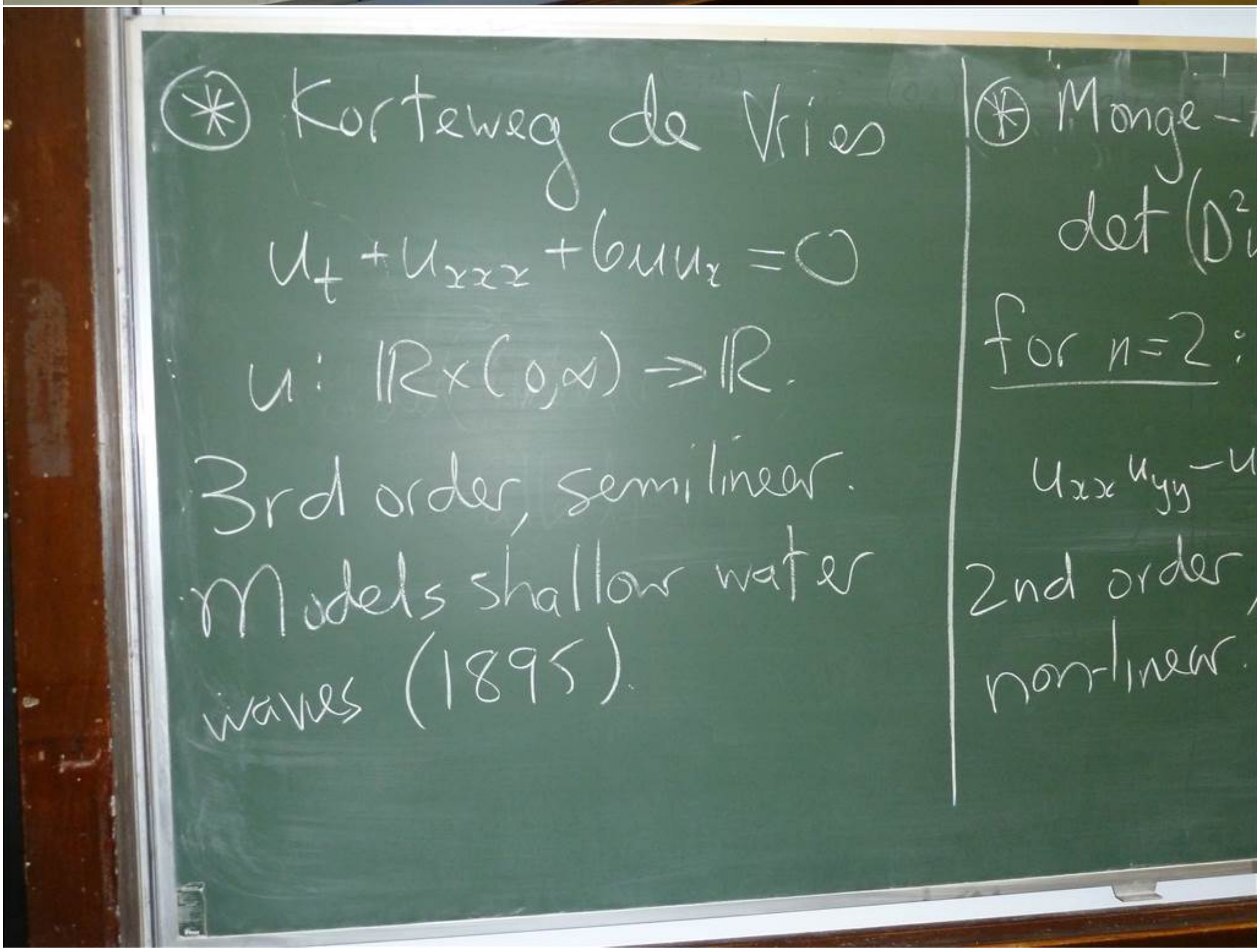

(H) $\begin{cases} u_t = \Delta u \\ + \text{initial/bdy cond}^{n}\text{s.} \end{cases}$
Examples: physical evolution models/ problems, e.g. heat flow, reaction/diffusion.
(H) is well posed for suitable Ω & u_0.
(W) $u_{tt} = \Delta$
$u(x,0) =$
$u_t(x,0) =$
(W) models:
n = 1: vibrat
u
snapshot a


(W) $\begin{cases} u_{tt} = \Delta u \text{ on } \mathbb{R}^n \times (0,\infty) \\ u(x,0) = u_0(x) \quad x \in \mathbb{R}^n \\ u_t(x,0) = g_0(x) \quad x \in \mathbb{R}^n \end{cases}$
(W) models:
n = 1: vibrating string.
u
x
snapshot at time t.
n = 2: vibratin
water waves
n = 3: Sound
(W) is well
Each of (D),
linear.
(D) prototypical
(H) " "
(W) " "


n=2 : vibrating membrane, surface water waves, earthquakes.
n=3 : Sound waves, light waves.
W is well posed for "nice" u_0 & g.
Each of D, W & H is 2nd order linear.
D prototypical elliptic eq^n.
H " " parabolic "
W " " hyperbolic "

Korteweg de Vries
$u_t + u_{xxx} + 6uu_x = 0$
$u: \mathbb{R}\times(0,\infty) \to \mathbb{R}$.
3rd order, semilinear.
Models shallow water waves (1895).
Monge-
det $(D^2$
for n=2:
$u_{xx}u_{yy} - u$
2nd order,
non-linear.

(*) Monge–Ampère

$$\det\left(D^2 u(x)\right) = f(x).$$

$D^2 u(x)$ → mx of 2nd partials

<u>for $n=2$:</u>

$$u_{xx}u_{yy} - u_{xy}^2 = f.$$

2nd order, fully non-linear.

Stuff — container

models mass transport (minimize a "cost functional".

Monge 1784, Ampère 1820.

Transport eq^n:

$$u_t + b u_x = 0 \quad (T)$$

b constant $\neq 0$.

$u : \mathbb{R} \times (0, \infty) \to \mathbb{R}$

models transfer of "stuff", e.g. heat, fluid, mass, information in 1-space dim.

Suppos… of (T). Define … z(s)=… z'(s)= … = … ⇒ z is a…

Suppose we have a sol^n of (T).

Define $z : \mathbb{R} \to \mathbb{R}$

$$z(s) = u(x + bs, t + s)$$

$$z'(s) = b u_x(x+bs, t+s) + u_t(x+bs, t+s)$$

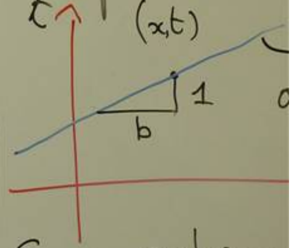
$$= 0 \text{ by } (T).$$

$\Rightarrow z$ is a constant f^n of s.

So given a…

u is constan…

slope $1/b$ th…

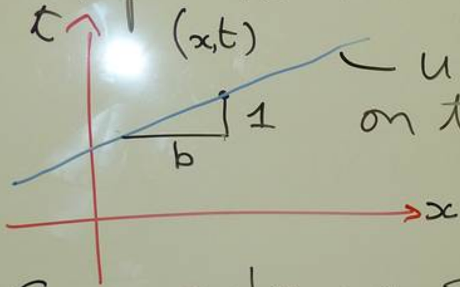


So, given…

initial valu…

problem

So given any $(x,t) \in \mathbb{R} \times (0,\infty)$, u is constant on the line of slope $1/b$ through (x,t).



— u is constant $= u(x,t)$ on this line.

So, given a solution of the initial value problem (I) $\begin{cases} \text{(T)} & \text{on } \mathbb{R}\times(0,\infty) \\ \underbrace{u(x,0) = g(x) \quad x \in \mathbb{R}}_{\text{initial condition}} \end{cases}$,

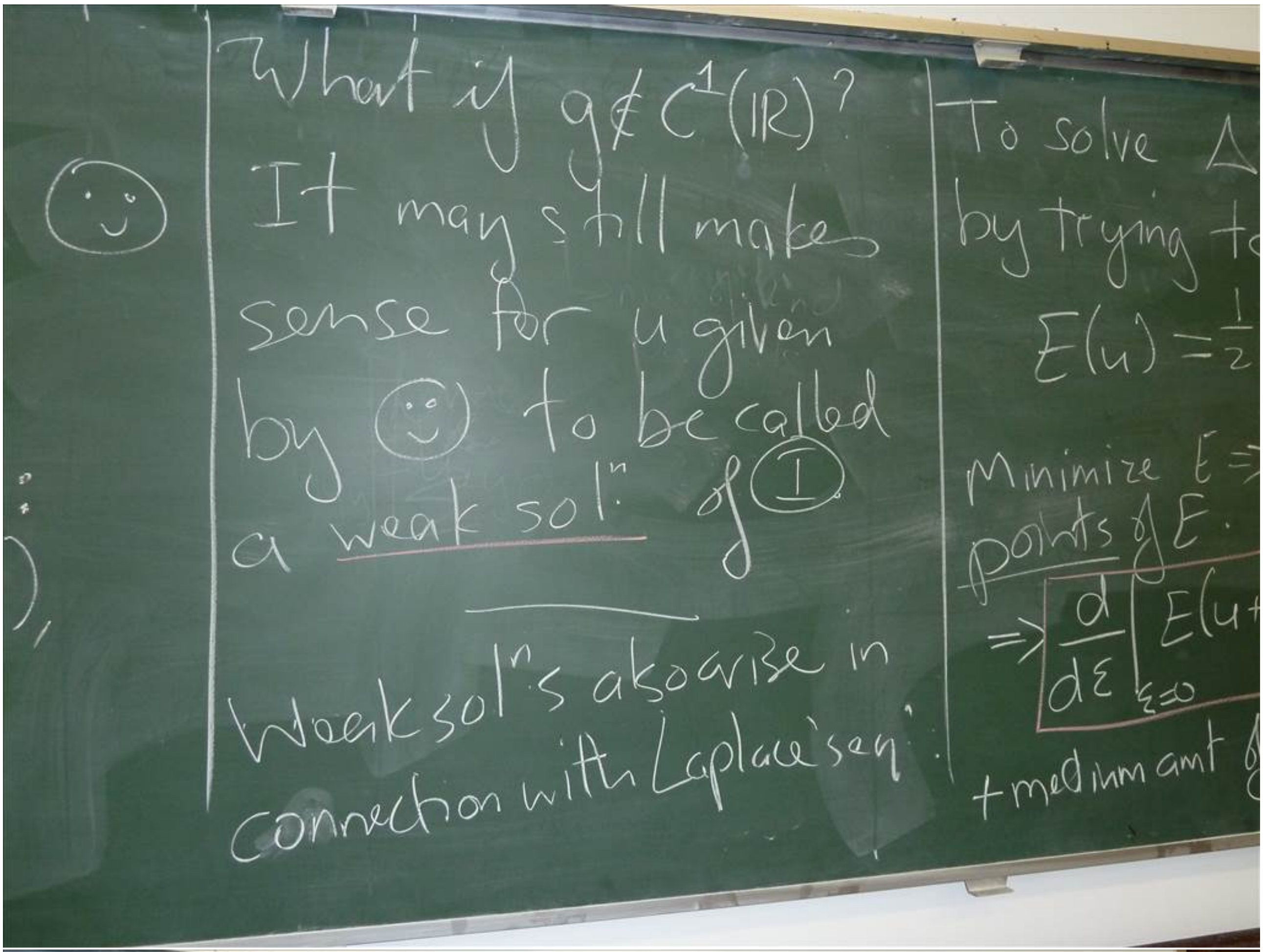
The above shows:

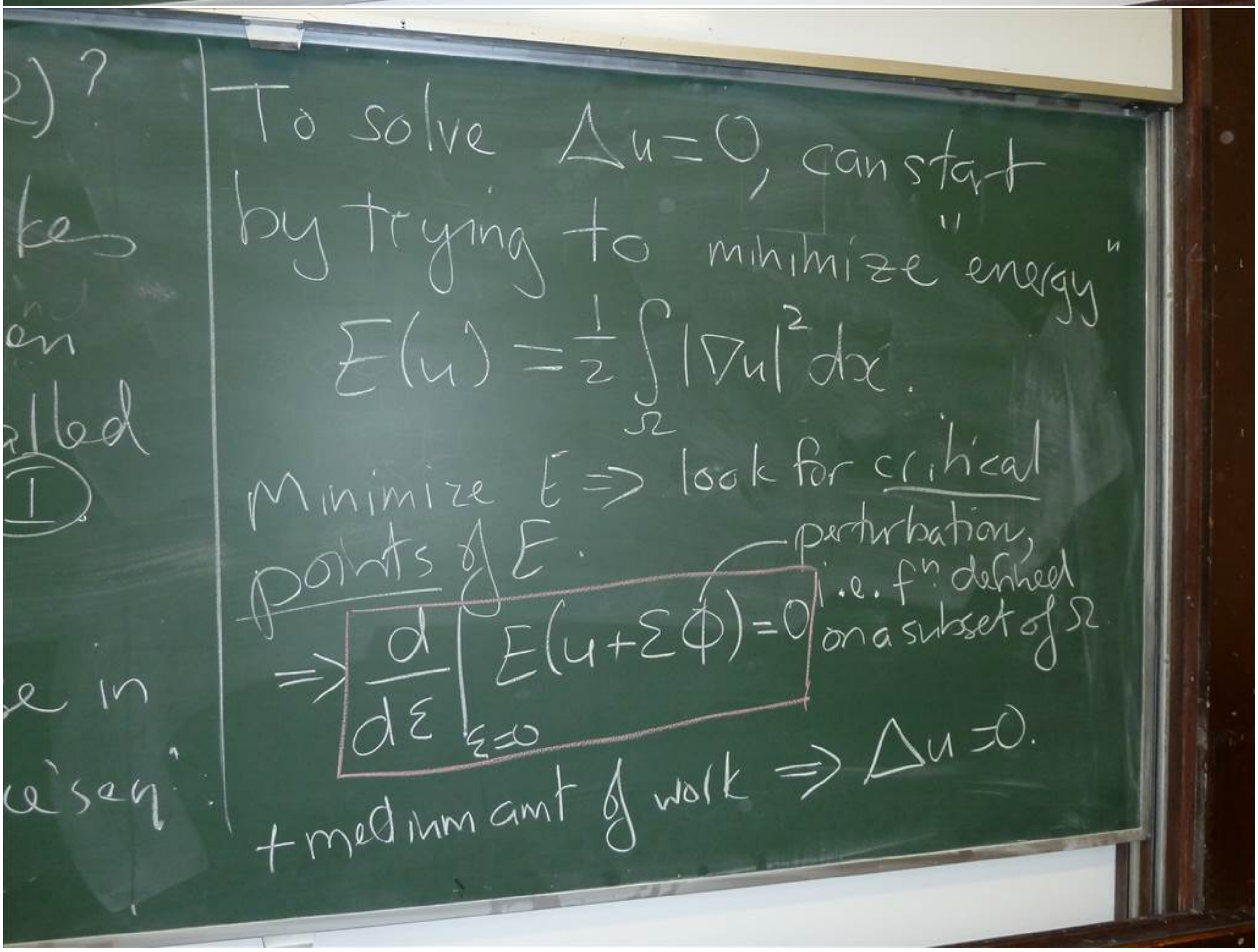
$$\boxed{u(x,t) = g(x-bt)} \quad ☺$$

Conversely, if we have $g \in C^1(\mathbb{R})$ (for example: can get away with less), then u given by ☺ solves (I).

What if $g \notin$ ...
It may sti...
sense for ...
by ☺ to ...
a weak sol^n ...

Weak sol^n's ab...
connection with ...


What if $g \notin C^1(\mathbb{R})$?
It may still make sense for u given by ☺ to be called a weak sol^n of (I)
Weak sol^n's also arise in connection with Laplace's eq^n:


To solve $\Delta u = 0$, can start by trying to minimize "energy"
$E(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 dx$.
Minimize $E \Rightarrow$ look for critical points of E.
$\Rightarrow \boxed{\frac{d}{d\varepsilon}\Big|_{\varepsilon=0} E(u+\varepsilon\phi)=0}$
perturbation, i.e. f^n defined on a subset of Ω.
+ medium amt of work $\Rightarrow \Delta u = 0$.