For a contradiction suppose G is a minimal counter-example containing the configuration:

Let K be the planar triangulation formed from G by removing v (and its incident edges) and adding an edge from r to g (or y to b)

Since K has
G it has a
4 colours. Thi
vertex colouring
only v is not
We may assum
coloured red, bl
there is a colour
Let $G_{r\text{-}g}$ be
$G_{b\text{-}y}$ "

Since K has fewer vertices than G it has a proper vertex colouring with 4 colours. This induces a partial proper vertex colouring of G with 4 colours where only v is not coloured.

We may assume that r, b, g, y are coloured red, blue, green, yellow (or else there is a colour available for v, a contradiction!

Let $G_{r\text{-}g}$ be the subgraph of G induced by vertices coloured red and green.

$G_{b\text{-}y}$ " " " " " " " blue and yellow.

Let H be the connected compo
If H does not contain g then we ca
Thus assume H contains g.
There is a red-green path fro
forms a cycle C.
Since exactly one of b and y
different connected components
Thus we can interchange the colo
colour v blue — a contradicti

Let H be the connected component of $G_{r\text{-}g}$ containing r.
If H does not contain g then we can interchange the colours in H and colour v red – a co
Thus assume H contains g.
There is a red-green path from r to g, which together with $[r,v,g]$ forms a cycle C.
Since exactly one of b and y is inside the cycle C, b and y are in different connected components of $G_{b\text{-}y}$.
Thus we can interchange the colours in the component containing b and colour v blue — a contradiction!

on!
vertices coloured red and green.
" " blue and yellow.

$u_t = u_{xx}$ for $u(x,t)$ on $\mathbb{R}\times(0,\infty)$ ①

$* \; u(x,t) = v(z)$, $z = x/\sqrt{t}$.

(i) show u solves ① $\Leftrightarrow$ v solves $v'' + \frac{z}{2}v' = 0$. ②

$$u_t = v' \cdot \frac{\partial z}{\partial t} = v' \cdot \frac{-x}{2t^{3/2}} \qquad \frac{\partial z}{\partial t} = \frac{-x}{2t^{3/2}} \qquad \frac{\partial z}{\partial x} = 1/\sqrt{t}$$

$$u_x = v' \frac{\partial z}{\partial x} = v'/\sqrt{t}$$

& $u_{xx} = v'' \cdot \frac{1}{t}$

$u_t = u_{xx} \Rightarrow \frac{-xv'}{2t^{3/2}} = \frac{v''}{t}$

$\times t\ (>0) \Rightarrow \frac{-x}{2\sqrt{t}}v' = v''$

$\Rightarrow v'' + \frac{z}{2}v' = 0$, ie

Since $t>0$, we can reve this argument to show ② $\Rightarrow$ ①.

$Rx(0,\infty)$ ①

②

& $u_{xx} = v'' \cdot \frac{1}{t}$

$u_t = u_{xx} \Rightarrow \frac{-xv'}{2t^{3/2}} = \frac{v''}{t}$

$\times t \ (>0) \Rightarrow \frac{-x}{2\sqrt{t}} v' = v''$

$\Rightarrow v'' + \frac{z}{2} v' = 0$, i.e. ②.

Since $t > 0$, we can revese this argument to show

② $\Rightarrow$ ①.

Show general sol^n of ②
is $v(z) = c\int_0^z e^{-s^2/4}\,ds + c'$.
$c, c' \in \mathbb{R}$. Make sure you show all $c, c' \in \mathbb{R}$ can occur.
$w = v'$: ② $\Rightarrow$
$w' + \frac{z}{2} w = 0$
$\frac{w'}{w} = \frac{-z}{2}$.

$= \frac{v''}{t}$

v''

i.e. ②.

vese

ow

Show general sol^n of ②

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$w = v'$: ② $\Rightarrow$

$w' + \frac{z}{2} w = 0$.

$\frac{w'}{w} = \frac{-z}{2}$. $\quad$ $w \neq 0$

integrate:

$\int \frac{dw}{w} = \int -$

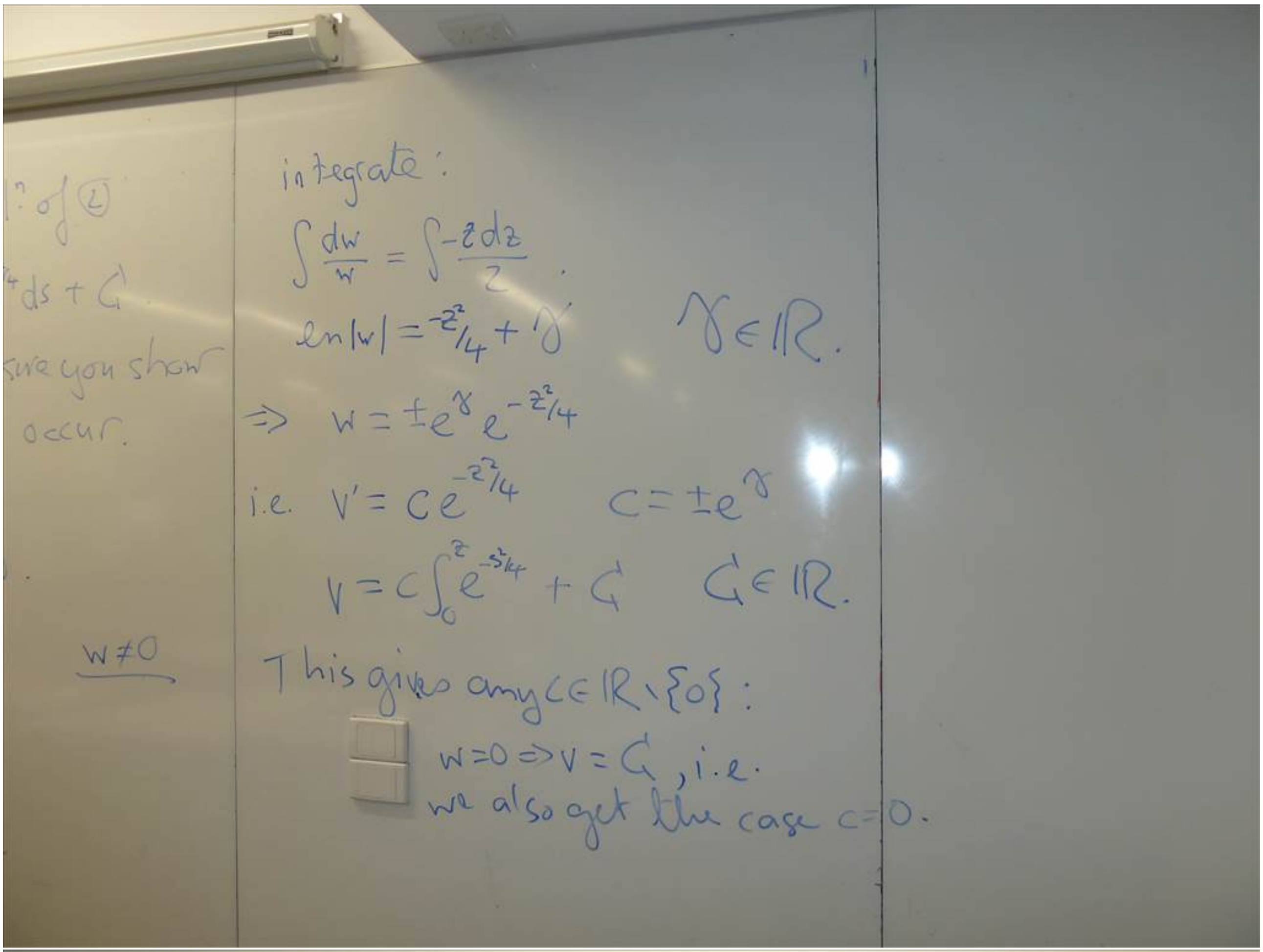
$\ln|w| = -z^2$

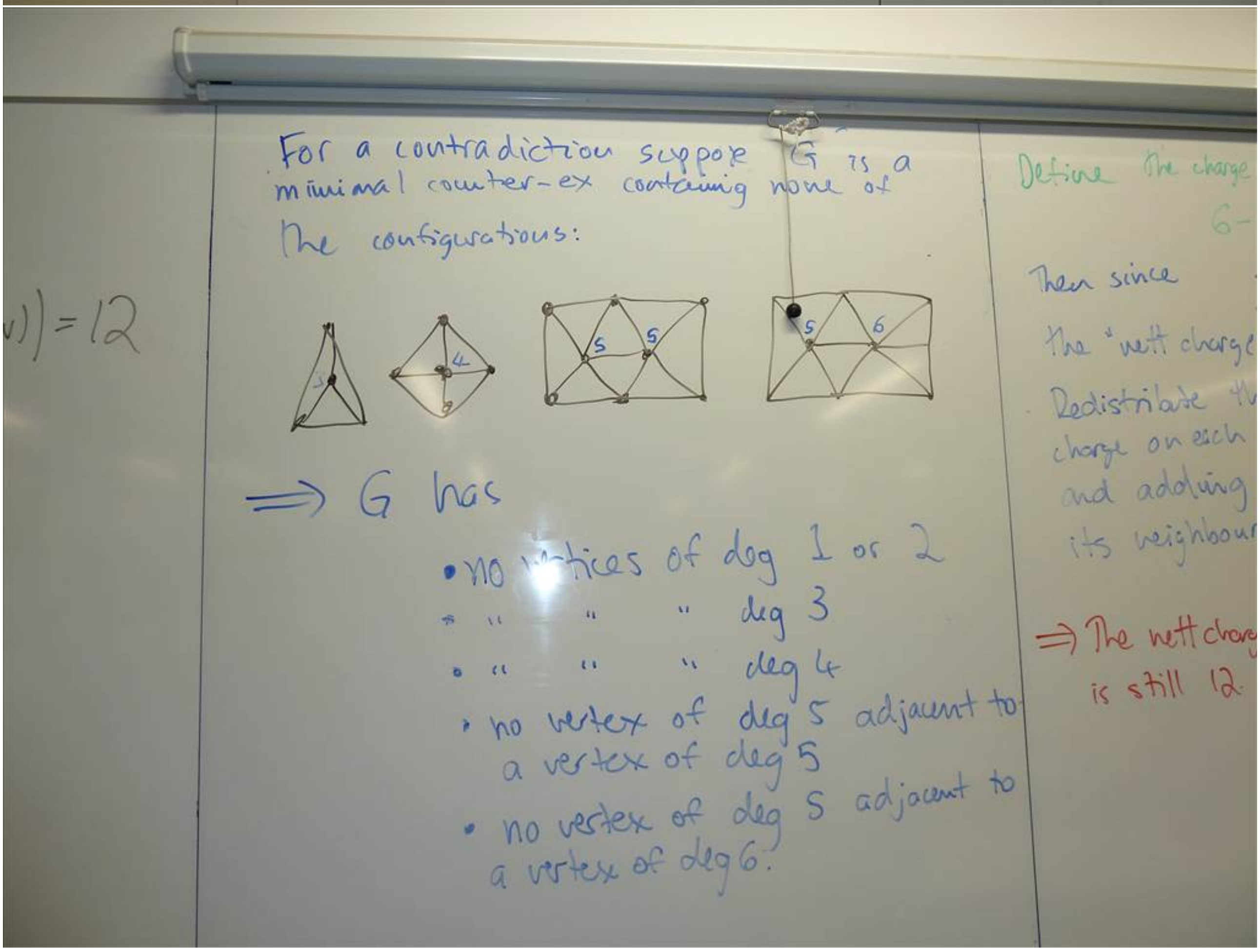
$\Rightarrow w = \pm e$

i.e. $v' = ce$

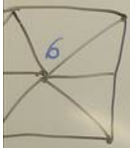
$v = c\int_0$

This give


1? of (2)
4ds + C1
sure you show
occur.
w ≠ 0
integrate:
$\int \frac{dw}{w} = \int \frac{-z\,dz}{2}$
$\ln|w| = -z^2/4 + \gamma$
$\gamma \in \mathbb{R}$.
$\Rightarrow w = \pm e^{\gamma} e^{-z^2/4}$
i.e. $v' = c e^{-z^2/4}$
$c = \pm e^{\gamma}$
$v = c\int_0^z e^{-s^2/4} + C_1$
$C_1 \in \mathbb{R}$.
This gives any $c \in \mathbb{R} \setminus \{0\}$:
$w = 0 \Rightarrow v = C_1$, i.e.
we also get the case $c = 0$.


v)) = 12
For a contradiction suppose G is a minimal counter-ex containing none of the configurations:
4
4
5 5
5 6
⇒ G has
• no vertices of deg 1 or 2
• " " " deg 3
• " " " deg 4
• no vertex of deg 5 adjacent to a vertex of deg 5
• no vertex of deg 5 adjacent to a vertex of deg 6.
Define the charge
6-
Then since
the "nett charge
Redistribute th
charge on each
and adding
its neighbour
⇒ The nett charg
is still 12.

is a
of



r 2

adjacent to

jacent to

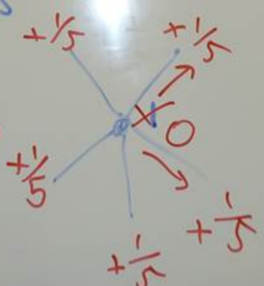
Define the charge on each vertex v in G by

$$6-\deg(v)$$

Then since $\sum_{v\in V(G)}(6-\deg(v))=12$

the "nett charge" on G is 12.

Redistribute the charge by reducing the charge on each vertex of deg 5 by 1 and adding a charge of $\frac{1}{5}$ to each of its neighbours



$\Rightarrow$ The nett charge is still 12.

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Since no vertices of deg 5 are adjacent so each vertex of deg 5 has charge 0

Since no vertex of deg 6 is adjacent to a vertex of deg 5, each vertex of deg 6 has charge 0.

If v is a vertex with $\deg(v) \geqslant 8$ Then its charge is at most

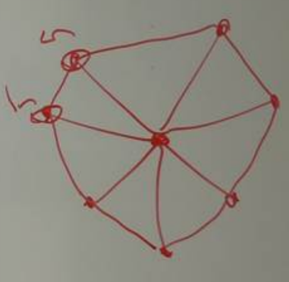
$$6-\deg(v)+\frac{\deg(v)}{5}$$

which is negative.

Thus the positive charge must be made up entirely from vertices of deg 7.

For a vertex of deg 7 to have positive charge it must have at least 6 neighbours of deg 5

(since $-1+\frac{x}{5} \leqslant 0$ for $x<6$)



Since G is a triangulation, this means that we have 2 adjacent vertices of deg 5

a contradiction!