

**MATH3404, Assignment 4 (Due on 31 Oct 2014)**  
**Submission: At the Assignment Box**  
**on Level 4 of the Priestley Building 67.**

**Question 1.** (3 marks) *Solve the problem of time-optimal control to the origin for the system*

$$\begin{aligned}\dot{x}_1 &= 3x_1 + x_2 \\ \dot{x}_2 &= 4x_1 + 3x_2 + u,\end{aligned}$$

where  $|u| \leq 1$ .

**Question 2.** (4 marks) *Solve the problem of time-optimal control to the origin for the system*

$$\dot{x}_1 = 2x_2, \quad \dot{x}_2 = -2x_1 + 4u, \quad \text{where } |u| \leq 1.$$

**Question 3.** (3 marks)

*The system  $\dot{x} = -x + u$ , where  $u = u(t)$  is not subject to any constraint, is to be controlled from  $x(0) = 1$  in such a way that*

$$J = \frac{1}{2}x^2(2) + \frac{1}{2} \int_0^2 (u^2 - 2xu) dt$$

*is minimized. Find the optimal control.*