MATH3404, Tutorial Problems 2 (week 3)

Question 1*. *Find the critical points of the following constrained optimization problems*

$$f(x_1, x_2, x_3) = x_1x_2 + x_2x_3 + x_3x_1 \quad \textit{subject to} \quad x_1 + x_2 + x_3 = 1$$

and check that they are non-degenerate. Determine the local minima and maxima.

Question 2*. *Find the local maxima and minima of the following problem by introducing two Lagrange multipliers*

$$f(x_1, x_2, x_3) = 2x_1 + 2x_2 + x_3$$

subject to $x_1^2 + 2x_2^2 + 4x_3^2 = 1$ *and* $(x_1 - 1)^2 + 2x_2^2 + 4x_3^2 = 2$.

Question 3. *The following constrained problem has four critical points, two of which are non-degenerate. Show that one of these is a local maximum and the other local minimum:*

$$f(x_1, x_2) = x_1^3 + x_2^3 + 3x_1^2 - 3x_2^2 - 8 \ \textit{subject to} \ \ x_1^2 + x_2^2 = 16.$$

(If f has a local minimum with constraints, we have the fundamental inequality

$$f(\mathbf{a} + \epsilon\mathbf{h}) - f(\mathbf{a}) \geq 0$$

for all tangent vector h to the constraint) Show that neither of the degenerate points can be maximum or a minimum.