MATH3404 Tutorial Sheet 9 (week 11)

Question 1. Solve the problem of time-optimal control to the origin for each of the following systems

$$
\begin{array}{rlll}
(i^*) & \dot{x}_1 = -3x_1 + 2x_2, & \dot{x}_2 = 2x_1 - 3x_2 + u, & \text{where } |u| \leq 1 \\
(ii) & \dot{x}_1 = x_2, & \dot{x}_2 = -3x_1 - 4x_2 + u, & \text{where } |u| \leq 1 \\
(iii) & \dot{x}_1 = x_2, & \dot{x}_2 = -x_2 + u, & \text{where } |u| \leq 1
\end{array}
$$

Question 2*. Let $\dot{x}_1 = x_2$ and $\dot{x}_2 = u$ where $|u| \leq 2$ (compare with that given in Example 2 from lectures).

(a) Find the time-optimal control from $(2, 2)$ to $(-2, 0)$ and calculate the minimum time. Find also the time-optimal control and minimum time from $(-2, 0)$ to $(2, 2)$. Comment!

(b) Suppose that the constraint is changed to $0 \leq t \leq 2$. Show that there is a time-optimal control to the origin only if $x_2 < 0$ and $x_1 \geq x_2^2/4$.

Question 3. The system $\dot{x}_1 = x_2$, $\dot{x}_2 = x_1 + u$, $|u| \leq 2$ is to be controlled from $\underset{\sim}{x}^0$ to $\underset{\sim}{x}^1$ in minimum time. Show that the time optimal control can only take the values $+2$ or -2 and that it can switch at most once. Given that $\underset{\sim}{x}^0 = (-1, 0)$ and $\underset{\sim}{x}^1 = (1, 0)$ show that the switch takes place at $(0, \sqrt{3})$ and find the time at which the switch takes place. Show that the minimum transfer time is $2 \sinh^{-1} \sqrt{3}$.

Question 4. Solve the problem of time-optimal control to the origin for the system

$$\dot{x}_1 = x_1 + x_2 + \alpha u, \quad \dot{x}_2 = 4x_1 + x_2 + u, \text{ where } |u| \leq 1 \tag{1}$$

in the cases (i) $\alpha = 0$ and (ii) $\alpha = -1/2$.