MATH3404 Tutorial Sheet 8 (week 9)

1*. Let $\dot{x}_1 = x_2 = f_1$ and $\dot{x}_2 = u = f_2$ where $|u| \leq K/m$. Suppose that the initial state is $\underset{\sim}{x}^0 = (x_1^0, x_2^0) = (a, b)$. Control the system to the origin in minimum time. Suppose the initial state (a, b) lies above the switching curve POQ and $t = \eta$ is the time at which the optimal control switches from $-K/m$ to K/m.

(i) Show that at $t = \eta$ the system is at
$(x_1^0, x_2^0) = (l, s)$ where

$$l = a + m(b^2 + 2Ka/m)/(4K), \quad s = -(2Kl/m)^{1/2} \tag{1}$$

(ii) Apply the condition $H = 0$ at $t = 0,\ t = \eta$ and $t = t_1$ and deduce that

$$\begin{aligned} A &= 1/s \qquad\qquad B = m(b - s)/(Ks), \\ \eta &= m(b - s)/K,\ t_1 = m(b - 2s)/K \end{aligned} \tag{2}$$

(iii) Calculate H as a function of t in the two time intervals $[0, \eta]$ and $[\eta, t_1]$. Hence verify that $H = 0$ for all t in $[0, t_1]$.

2*. The system $\dot{x}_1 = -x_1 + u$, where $u = u(t)$ is not subject to any constraint, is to be controlled from $x_1(0) = 1$ to $x_1(t_1) = 2$ where t_1 is unspecified, in such as way that

$$J = \frac{1}{2}\int_0^{t_1} \left(x_1^2 + u^2\right) d\,t \tag{3}$$

is minimized. Find the optimal control.

3. The system $\dot{x}_1 = x_1 + u$, where $u = u(t)$ is not subject to any constraint, is to be controlled from $x_1(0) = a$ to $x_1(t_1) = b$ where t_1 is unspecified, in such as way that

$$J = \frac{1}{2}\int_0^{t_1} \left(2x_1^2 + 2ux_1 + u^2\right) d\,t \tag{4}$$

is minimized. Find the optimal control.