MATH3404 TUTORIAL SHEET 6 (Week 8)

Question 1*. *Show that $x = t/2$ is an extremal for*

$$J[x] = \int_0^2 (\dot{x}^2 - 1)^2 d\,t, \qquad x(0) = 0,\ x(2) = 1.$$

Find the corresponding value of J. Find a piecewise smooth curve with $J = 0$. You may find this curve by inspection or otherwise but you must check that the corner conditions are satisfied.

Question 2*. *Find a suitable field of extremals for the following problem:*

$$\int_1^2 (\dot{x} + t^2\dot{x}^2)\, dt,\ \ x(1) = 0,\ x(2) = 1. \tag{1}$$

Show that the extremal is a strong (local) minimizing curve (by the Weierstrass's sufficient Theorem).

Question 3. *Find a suitable field of extremals for the following problem:*

$$\int_0^1 (\frac{1}{2}\dot{x}^2 + x\dot{x} + x + \dot{x})\, dt,\ \ x(0) = 0,\ x(2) = 2. \tag{2}$$

Show that the extremal is a strong (local) minimizing curve (by the Weierstrass's sufficient Theorem).