MATH3404 TUTORIAL SHEET 5 (Week 7)

To encourage your participation, the course coordinator introduce a peer assessment in tutorials.

Question 1*. *By considering an admissible variation* $y = x^*(t) + \eta(t)$*, where* $\eta(t)$ *is* C^1 *and not necessarily small, show that the extremals found in Tutorial Problem 2 : 1.) (i) and (iii) are minimizing curves: i.e. for*

(i) $$\int_1^2 \frac{\dot{x}^2}{t^3} \, dt \quad with \ x(1) = 2, \, x(2) = 17.$$

(iii) $$\int_0^\pi (\dot{x}^2 + 2x \sin t) \, dt \quad with \ x(0) = x(\pi) = 0.$$

Question 2. *Find the extremals of* $\int_0^\pi \dot{x}^2 \, dt$ *with* $x(0) = 0$*,* $x(\pi) = 0$ *subject to* $\int_0^\pi x^2 \, dt = \frac{\pi}{2}$*. Show that there is an infinite set of extremals. Evaluate the functional on a typical extremal.*

Question 3. *Find the extremal of* $\int_0^2 (\dot{x}^2 + x) \, dt$*, where* $x(0) = 1$*,* $x(2) = 48$ *subject to the constraint* $\int_0^2 tx \, dt = 43$*.*