

MATH3404, Tutorial problems 4 (Week 6)

Find the extremals for each of the following:

1.)

$$\int_0^1 (x^2 + \dot{x}^2 + 2xe^t) dt \quad \text{with } x(0) = 0, x(1) \text{ free.}$$

2.)

$$\int_1^2 (\dot{x} + t^2 \dot{x}^2) dt \quad \text{when } x(1) = 0, x(2) \text{ free.}$$

3.)

$$\int_0^1 \left(\frac{1}{2} \dot{x}^2 + x\dot{x} + x + \dot{x} \right) dt \quad \text{when } x(0), x(1) \text{ are both free.}$$

4*.)

$$\int_0^T \frac{\dot{x}^2}{t^3} dt$$

when $x(0) = 1$ and $x(T)$ lies on the curve $x = 2 + (t - 1)^2$.

5.)

$$\int_0^T \frac{(1 + \dot{x}^2)^{\frac{1}{2}}}{x} dt$$

when $x(0) = 0$ and $x(T) > 0$ lies on the curve $x = t - 5$.