MATH3404, Tutorial problem 3 (at Week 5)

Question 1. *Find the extremal for each of the following fix-end point problems:*

(i*) $\displaystyle\int_1^2 \frac{\dot{x}^2}{t^3}\,dt$ *with* $x(1) = 2$, $x(2) = 17$.

(ii) $\displaystyle\int_0^{\frac{\pi}{2}} (x^2 - \dot{x}^2 - 2x\sin t)\,dt$ *with* $x(0) = 1$, $x(\frac{\pi}{2}) = 2$.

(iii) $\displaystyle\int_0^{\pi} (\dot{x}^2 + 2x\sin t)\,dt$ *with* $x(0) = x(\pi) = 0$.

Question 2. *Find the extremal for each of the following:*

(i*) $\displaystyle\int_0^2 \frac{\dot{x}^2}{x^3}\,dt$ *with* $x(0) = 1$, $x(2) = 4$.

(ii) $\displaystyle\int_0^2 (\frac{1}{2}\dot{x}^2 + x\dot{x} + x + \dot{x})\,dt$ *with* $x(0) = 0$, $x(2) = 2$.

(iii) $\displaystyle\int_0^1 \frac{(1+\dot{x}^2)^{\frac{1}{2}}}{x}\,dt$ *with* $x(0) = 0$, $x(1) = \sqrt{3}$.