

MATH3404 Tutorial Sheet 10 (week 12)

1*. Solve the problem of time-optimal control to the origin for the following systems

$$\dot{x}_1 = 3x_1 + x_2, \quad \dot{x}_2 = -x_1 - 2x_2 + u, \quad \text{where } |u| \leq 2;$$

2. The system $\dot{x} = -\alpha x + u$, $|u| \leq 2$, is to be controlled from $x(0) = 0$ to $x(2) = b$ (where α and b are given) in such a way as to minimize $-x(2)$. Find the time optimal control.

3. The oscillatory system $\dot{x}_1 = x_2$, $\dot{x}_2 = -\pi^2 x_1 + u$, is to be controlled with unconstrained control u in the time interval $0 \leq t \leq 1/2$ from $x_1(0) = 0$, $x_2(0) = 1$ so that

$$J = [x_1(1/2)]^2 + \int_0^{1/2} u^2 dt$$

is minimized. Find the optimal control and the corresponding value of J . is to find the optimal control.