MATH3404 TUTORIAL SHEET 6 (Week 8)

Question 1*. *Find a suitable field of extremals for the following problem:*

(1) $$\int_1^2 (\dot{x} + t^2\dot{x}^2)\, dt, \;\; x(1) = 0, \;\; x(2) = 1.$$

Show that the extremal is a strong (local) minimizing curve (by the Weierstrass's sufficient Theorem).

Question 2*. *Find a suitable field of extremals for the following problem:*

(2) $$\int_0^1 (\frac{1}{2}\dot{x}^2 + x\dot{x} + x + \dot{x})\, dt, \;\; x(0) = 0, \;\; x(2) = 2.$$

Show that the extremal is a strong (local) minimizing curve (by the Weierstrass's sufficient Theorem).

Question 3.

(A) Let

$$J(x) = \int_{t_0}^{t_1} \left(s(t)\dot{x}^2 + q(t)x^2\right) dt,$$

where $s(t)$ and $q(t)$ are C^1. Show that the extremals of J must satisfy

(3) $$\frac{d}{dt}\left(s(t)\dot{x}\right) = q(t)x.$$

Show that if $s(t)$ and $q(t)$ are positive on $[t_0, t_1]$ with $x(t_0) = x_0$ and $x(t_1) = x_1$, then any curve $x = \phi(t)$ satisfying equation (3) and passing through the end - points gives J a strong local minimum by the Weierstrass's sufficient Theorem. Show that the minimum value of J is

$$J_{\min} = [s(t)\phi(t)\dot{\phi}(t)]_{t_0}^{t_1}.$$

(B) to find the minimizing curve and the value of $J_{\min}$ for the cases

$$\int_1^2 \left(\dot{x}^2 + \frac{2}{t^2}x^2\right) dt, \;\; \textit{where } x(1) = 1, \;\; x(2) = 4,$$

(This is a very challenging question.)