Question 1. (3 Marks)
Find the local minimum of the function
\[ f(x_1, x_2, x_3) = x_1^2 - 2x_1x_2 + 2x_2^2 + x_3^2 \]
in \( \mathbb{R}^3 \).

Question 2. (3 Marks)
Find the critical points of the following constrained optimization problem
\[ f(x_1, x_2, x_3) = x_1^2 + x_2^2 + 2x_3^2 \quad \text{subject to} \quad g(x_1, x_2, x_3) = x_1 + x_2 + x_3 = 4 \]
and check that they are non-degenerate. Determine the local minima and maxima.

Question 3. (4 Marks)
Find the local maxima and minima of the following problem by introducing two Lagrange multipliers:
\[ f(x_1, x_2, x_3) = 2x_1 + x_2 + x_3 \]
subject to \( x_1^2 + x_2^2 + x_3^2 = 2 \) and \( x_1^2 + (x_2 - 1)^2 + x_3^2 = 3 \).