1. (3 marks) Use the Weierstrass condition, find the (Strongly) minimizing curve and the value of $J_{\text{min}}$ for the cases

$$\int_1^2 (t^2 \dot{x}^2 + 2x^2) dt$$

where $x(1) = 0, x(2) = 7$.

2. (4 marks) The system $\dot{x} = x + u$, where $u = u(t)$ is not subject to any constraint, is to be controlled from $x_1(0) = 3$ to $x_1(t_1) = 2$ where $t_1$ is unspecified, in such a way that

$$J = \int_0^{t_1} \left( x^2 + ux + \frac{1}{2}u^2 \right) dt$$

is minimized. Find the optimal control.

3. (3 marks) Solve the problem of time-optimal control to the origin for the system $\dot{x}_1 = -3x_1 + x_2$ and $\dot{x}_2 = x_1 - 3x_2 + u$ where $|u| \leq 1$. 