MATH3404, Assignment 2 (due on 19 September 2014)
Submission: At the tutorial or at the Assignment Box on Level 4 of the Priestley Building 67.

Question 1. *(2 Marks)*
Find the extremal for the following:

$$\int_1^2 \frac{2t\dot{x}+\dot{x}^2}{t^2}\,dt \quad \textit{with } x(1)=1.5,\ x(2)=14$$

Question 2. *(2 Marks)*
Find the extremal for

$$\int_0^T (\dot{x}^2+2x\dot{x}+x^2)\,dt$$

with $x(0)=1$ *and for* $T>0$, $x(T)$ *lies on a given curve* $x=c(t)=3$.

Question 3. *(3 Marks)*
Find the extremals of $\int_0^1 [\frac{1}{2}\dot{x}^2+2x]\,dt$ *with* $x(0)=0$ *and* $x(1)=5$ *subject to the constraint* $\int_0^1 6tx\,dt=\frac{11}{2}$.

Question 4. *(3 Marks) Let* $x=x(t):[t_0,t_1]\to\mathbb{R}$ *be a curve in* C^2 *with boundary conditions* $x(t_0)=x_0$ *and* $x(t_1)=x_1$. *Consider a functional*

$$J[x]=\int_{t_0}^{t_1}[a(t)\dot{x}^2+b(t)x^2]\,dt,$$

where $a(t)$ *is a* C^2*-function and* $a(t)\geq 1$ *and* $b(t)\geq 0$ *for all* $t\in[t_0,t_1]$.

Assume that $x^*=x^*(t)$ *be an extremal for* $J[x]$ *and* x^* *also satisfies the boundary conditions* $x^*(t_0)=x_0$ *and* $x^*(t_1)=x_1$. *Prove that* $x^*=x^*(t)$ *must be a minimizer of* $J[x]$ *for all* C^2*-curve* $x(t_0)=x_0$ *and* $x(t_1)=x_1$.

Hints:
i) Let $y=x^*+\eta$ for any $\eta\in C^2[t_0,t_1]$ with $\eta(t_0)=\eta(t_1)=0$.
ii) compute

$$\Delta J=J[y]-J[x^*].$$

iii) Using integration by parts, you can show that $\Delta J\geq 0$.