MATH 3402

TUTORIAL SHEET 9

1. If

$$A = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$$

show that

$$||A||_2 = \frac{1}{2}\left(|b| + \sqrt{|b|^2 + 4|a|^2}\right) \ ,$$

and if

$$A = \begin{pmatrix} 0 & 1 \\ a & b \end{pmatrix}$$

then

$$||A||_2 = \frac{1}{2}\left(\sqrt{(|a|+1)^2 + |b|^2} + \sqrt{(|a|-1)^2 + |b|^2}\right) \ .$$

2. If $C(0,1)$ is the set of functions continuous on $[0,1]$ with the uniform metric, and $D(0,1)$ is the set of continuously differentiable functions on $[0,1]$ with the same metric;

(a) Is $T : C \to D$ given by $T(f)(x) = \int_0^x f(t)\,dt$ continuous?

(b) Is $T : D \to C$ given by $T(f)(x) = f'(x)$ continuous?

3. Let T be a linear transformation from ℓ^1 to ℓ^1.
Set $e_i = \{\delta_{ij}\}$ and $a_i = T(e_i)$.
Show that $||T|| = \sup_i ||a_i||_1$.

4. Let X be a finite dimensional normed linear space, and Y a normed linear space.
If T is a linear operator from X to Y, show that T is continuous.