MATH 3402

TUTORIAL SHEET 8

1. Which of the following are contractions on $\mathbb{R}^2$ with the Euclidean metric?

(i) $f:\ R^2 \rightarrow R^2$ given by the matrix $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

(ii) $f:\ R^2 \rightarrow R^2$ given by the matrix $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$

(iii) $f:\ R^2 \rightarrow R^2$ given by the matrix $\begin{pmatrix} \frac{1}{2} & \frac{6}{7} \\ -\frac{5}{6} & \frac{2}{3} \end{pmatrix}$

2. Show that if $h \in C(a,b)$ (with the uniform metric) and $b-a<1$, then $\mathcal{F} : C(a,b) \rightarrow C(a,b)$ is a contraction mapping, where

$$\mathcal{F}(g)(x) = h(x) + \int_a^x g(t)\,dt$$

for $x \in [a,b]$, $g \in C(a,b)$.

What is the fixed point of $\mathcal{F}$.

3. The space ℓ^1 is defined to be the set of all sequences $x = \{\xi^{(i)}\}$ such that $\sum_{i=1}^{\infty} |\xi^{(i)}|$ converges.

If $b = \{\beta^{(i)}\}$ and $c = \{\gamma^{(i)}\}$ are elements of ℓ^1, show that if $|\lambda|$ is sufficiently small, there is a unique element $a = \{\alpha^{(i)}\} \in \ell^1$ such that

$$\alpha^{(n)} = \beta^{(n)} + \lambda \sum_{i=1}^{\infty} \gamma^{(i)} \alpha^{(n+i-1)} .$$

4. Let T be the linear transformation from X to Y whose matrix representation is

$$T(x) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} .$$

Determine $||T||$ when

(a) $X = \ell^2(2)$ and $Y = \ell^2(2)$;

(b) $X = \ell^1(2)$ and $Y = \ell^1(2)$;

(c) $X = \ell^\infty(2)$ and $Y = \ell^\infty(2)$;

(d) $X = \ell^1(2)$ and $Y = \ell^\infty(2)$;