MATH 3402

TUTORIAL SHEET 5

1. If f is a many-one transformation of A into B, and A_1 and A_2 are subsets of A, prove that

(a) $$f(A_1 \cup A_2) = f(A_1) \cup f(A_2) \ ;$$

(b) $$f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2) \ .$$

In the second case, show that equality holds for all A_1 and A_2 if and only if f is a one-one transformation.

2. Let A be the set of real numbers, and let a subset of A be called open if it is A or the null set or if it consists of points x such that $x > k$ for some k.

Prove that the open sets defined in this way form a topology for A.

3. If $M_1 = (A_1, d_1)$ and $M_2 = (A_2, d_2)$ are two metric spaces, show that the function defined by

$$d\left((x_1, x_2), (y_1, y_2)\right) = d_1(x_1, y_1) + d_2(x_2, y_2)$$

where $x_1, y_1 \in A_1$ and $x_2, y_2 \in A_2$ is a metric on $A_1 \times A_2$.

Show that the topology generated by this metric is the product topology.

4. If S, T are topological spaces homeomorphic respectively to S', T', prove that $S \times T$ is homeomorphic to $S' \times T'$.