# MATH 3402
## Tutorial Sheet 4

1. Show that the union of two compact sets is compact, and that the intersection of any number of compact sets is compact.

2. List all possible topologies on

(i) $\qquad\qquad\qquad\qquad\qquad\qquad$ $\{a, b\}$

(ii) $\qquad\qquad\qquad\qquad\qquad\qquad$ $\{a, b, c\}$

3. Prove that any map $f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is continuous if either $\mathcal{T}_1$ is the discrete topology or $\mathcal{T}_2$ is the indiscrete topology.

4. Let $(X, d)$ be the set $\mathbb{Q}$ with the usual metric.
Show that the set $S = \{x \in \mathbb{Q}; x^2 < 2\}$ is both open and closed in $(X, d)$.

5. Prove that $f : (X, \mathcal{T}) \to \mathbb{R}$ is continuous if and only if for every $a \in \mathbb{R}$, $f^{-1}((-\infty, a))$ and $f^{-1}((a, \infty))$ are in $\mathcal{T}$ .